The University of Texas at Austin
Electrical and Computer Engineering
Cockrell School of Engineering

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INTRODUCTION TO COMPUTER VISION

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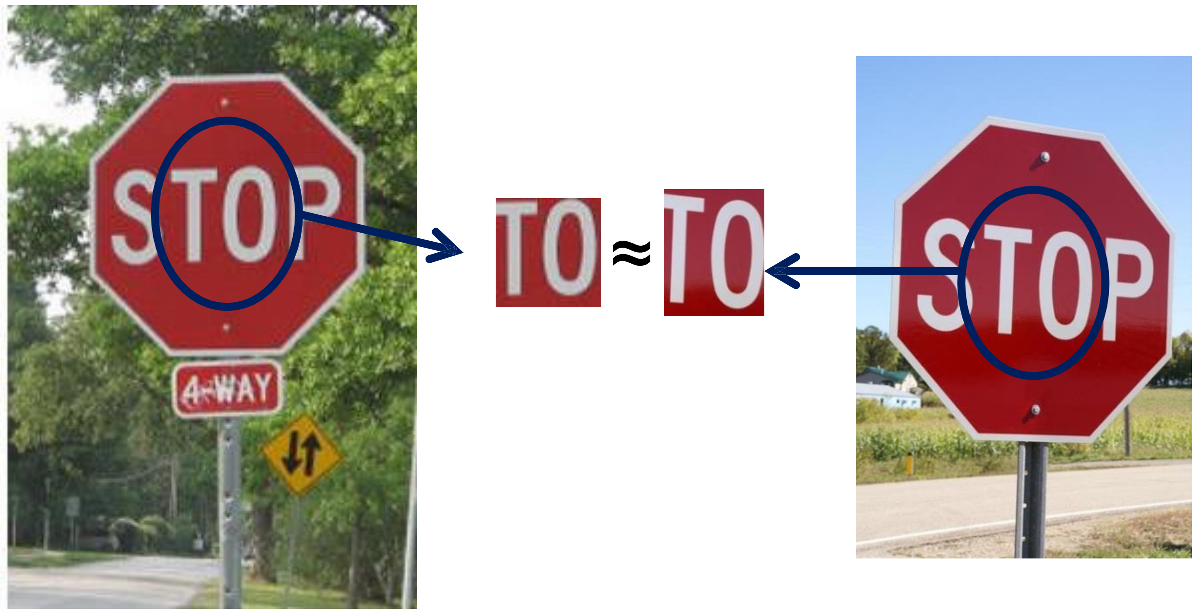
Famous tale in computer vision

- Once, a CMU graduate student asked the famous computer vision scientist **Takeo Kanade**: *"What are the three most important problems in computer vision?"*
- Takeo replied: "**Correspondence, correspondence, correspondence**!"

Visual Correspondence across views

Matching points, patches, edges, or regions across images.

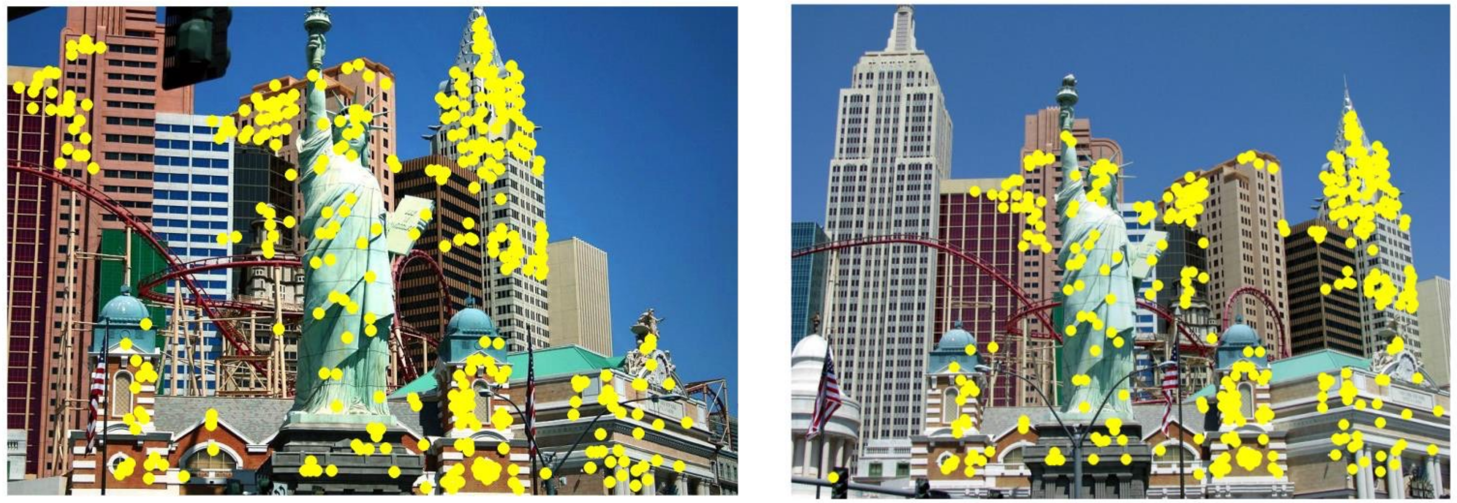
- *Sparse or local correspondence* (picking some “keypoints”)
- *Dense correspondence* (at every pixel)



Hays

Fundamental to Applications

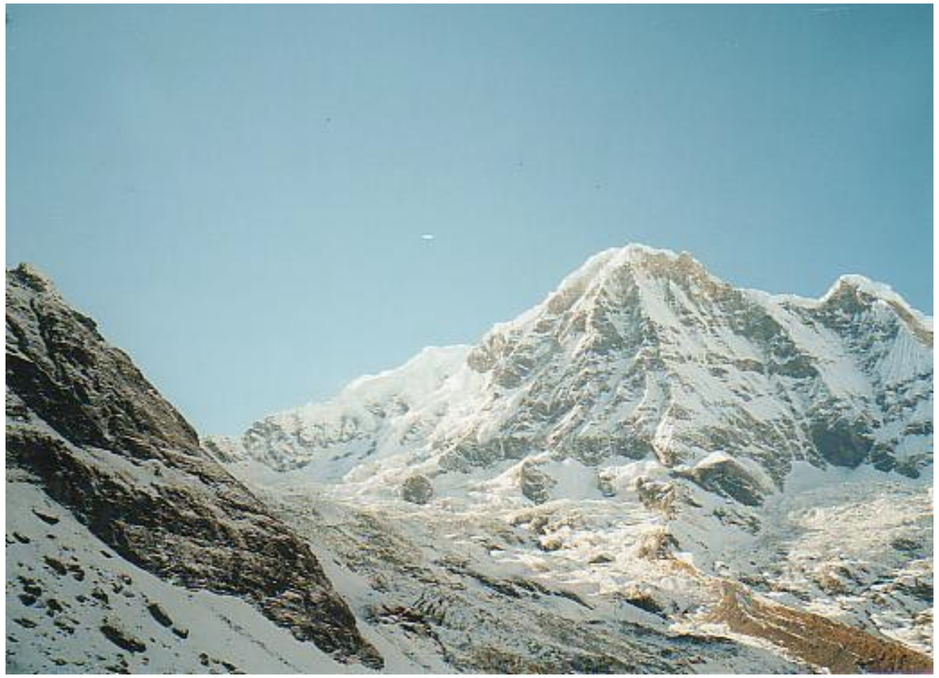
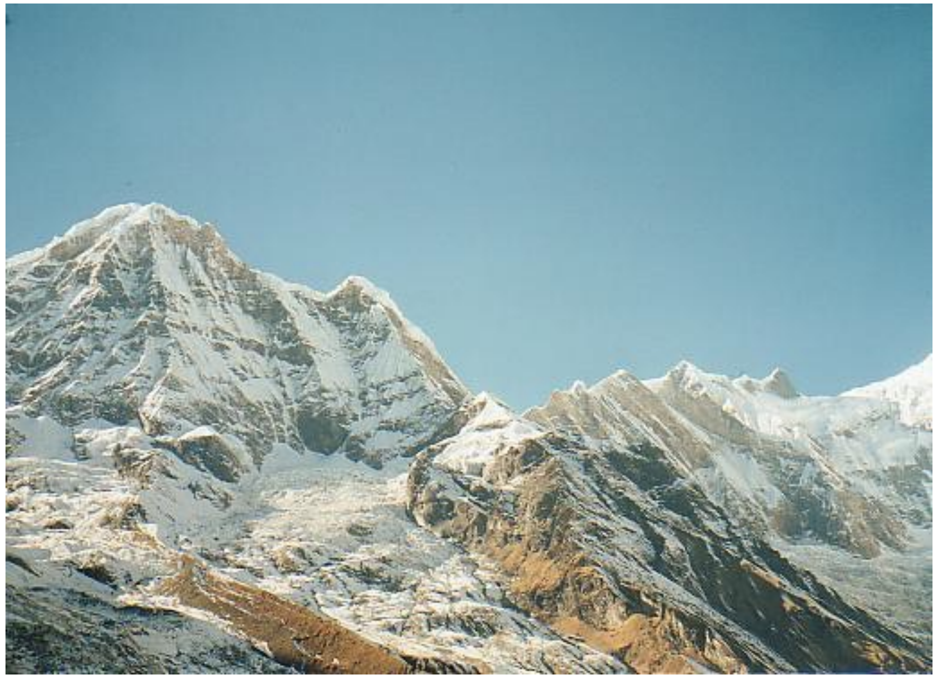
- Image alignment
- 3D reconstruction
- Motion tracking (robots, drones, AR)
- Indexing and database retrieval
- Object recognition



Hays

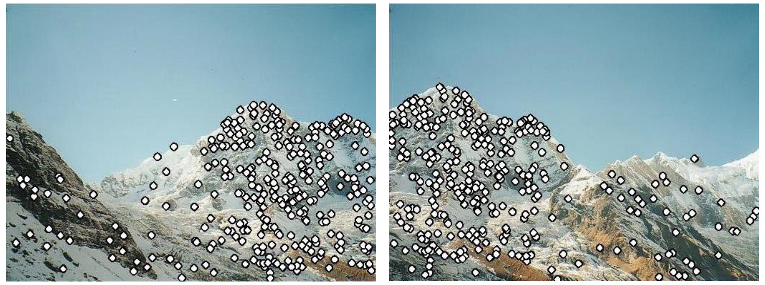
Example application: Panorama stitching

We have two images –
how do we estimate how to overlay them?

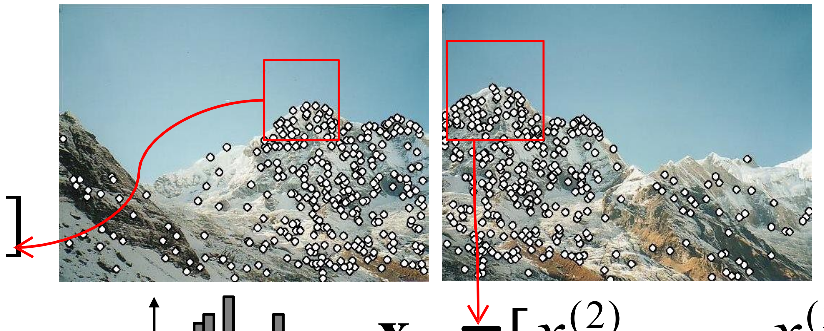



Local features: main components

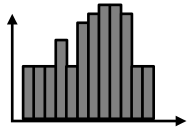
1) **Detection:**
   Find a set of distinctive key points.



2) **Description:**
   Extract feature descriptor around each interest point as vector.

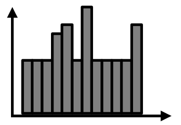


x_1



$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$

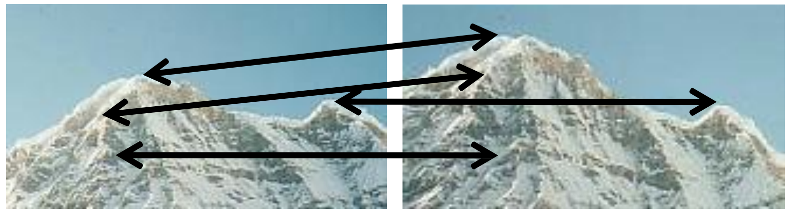
x_2



$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) **Matching:**
   Compute distance between feature vectors to find correspondence.

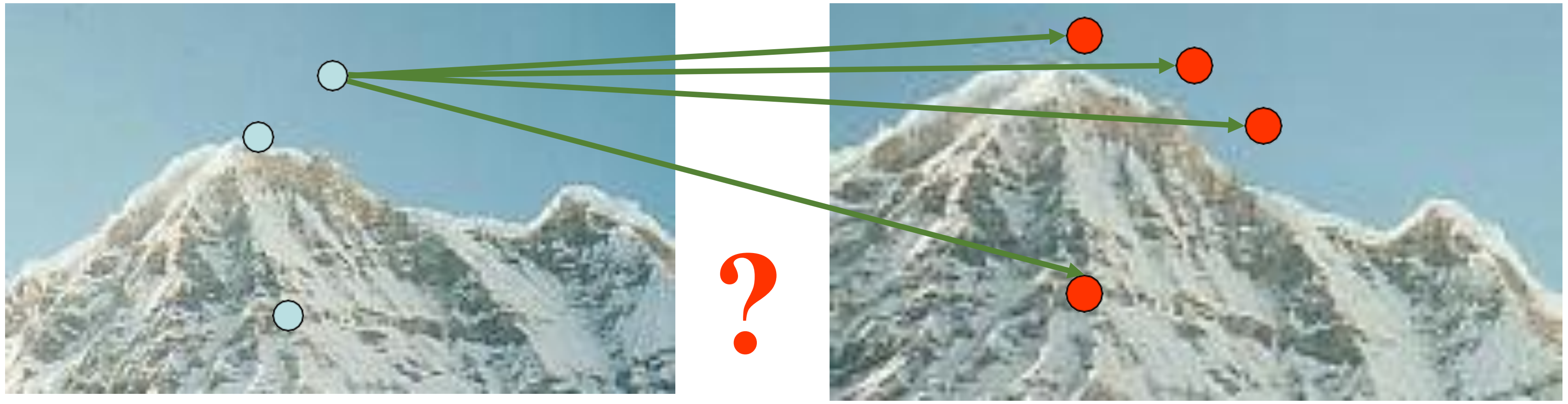
$$d(\mathrm{x}_1, \mathrm{x}_2) < T$$



K. Grauman, B. Leibe

Goal: Distinctiveness

We want to be able to reliably determine which point goes with which.



May be difficult in structured environments with repeated elements!

Kristen Grauman

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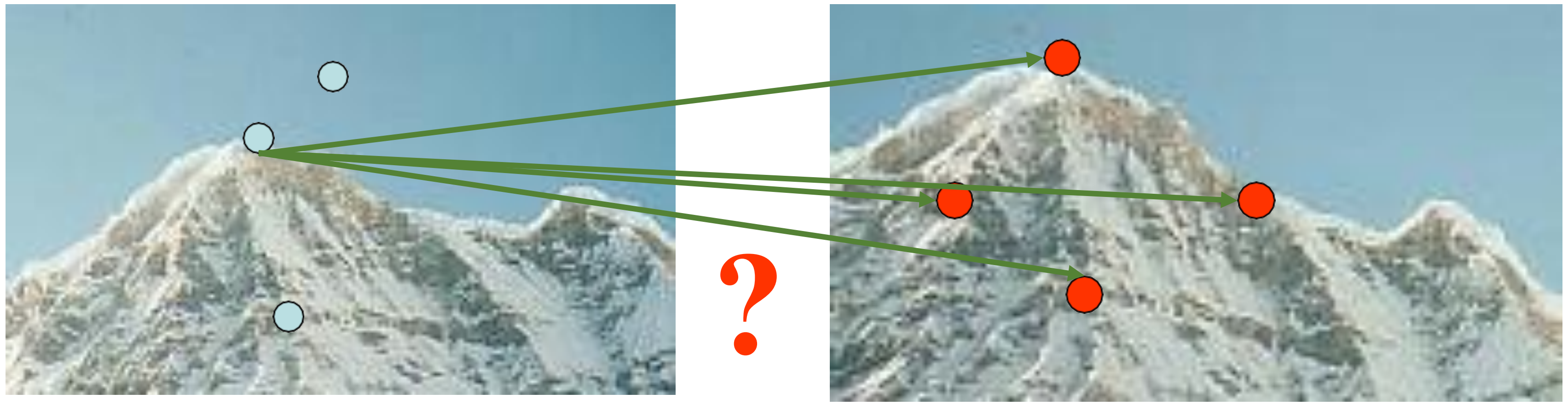

May be difficult in structured environments with repeated elements!

Kristen Grauman

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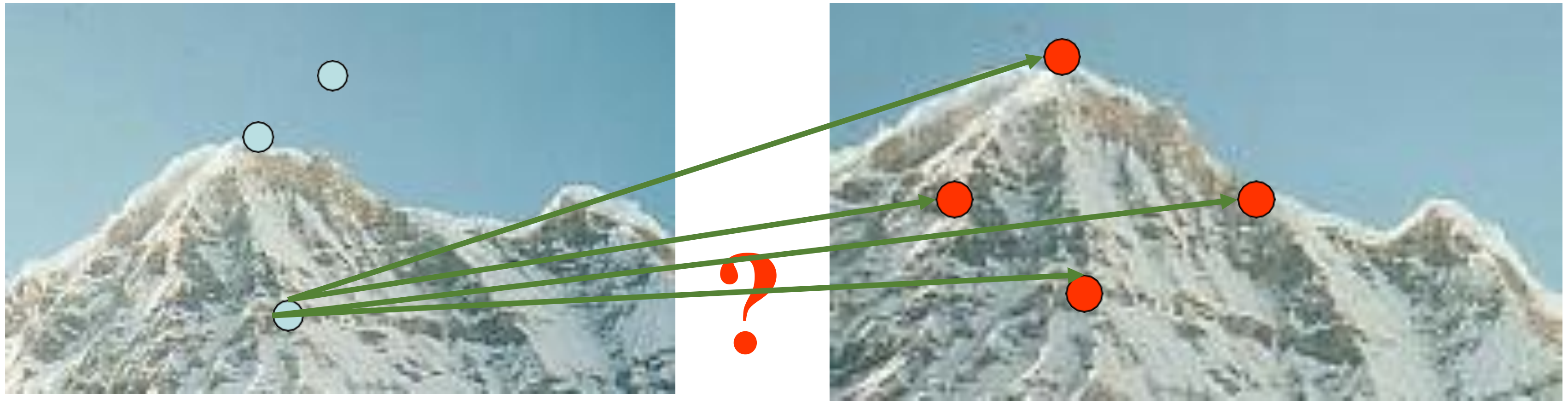
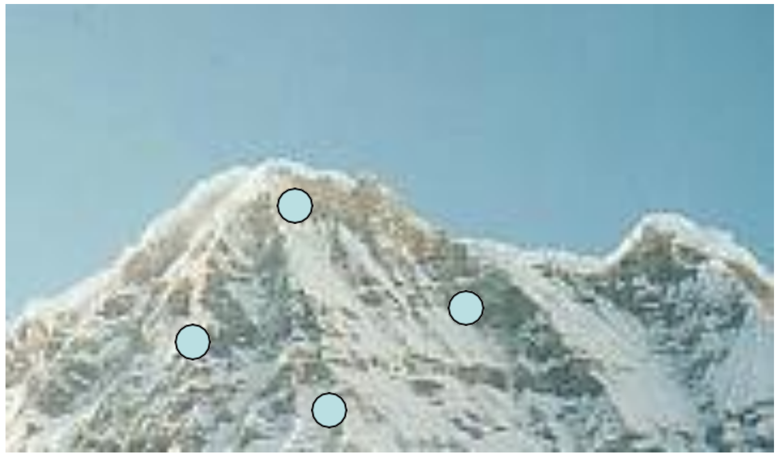


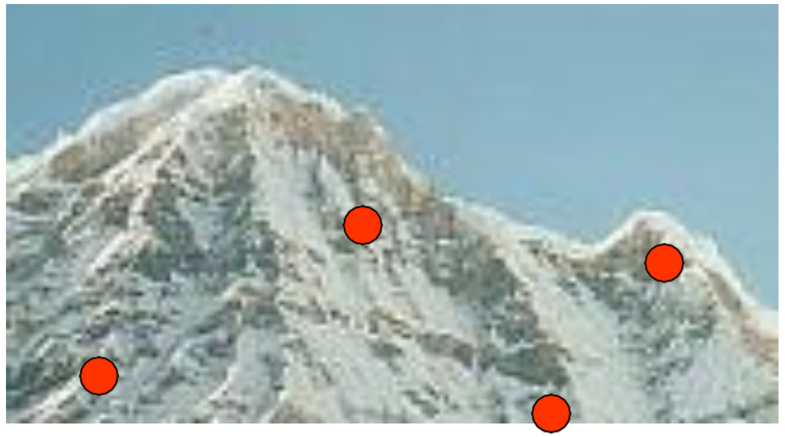
May be difficult in structured environments with repeated elements!

Kristen Grauman

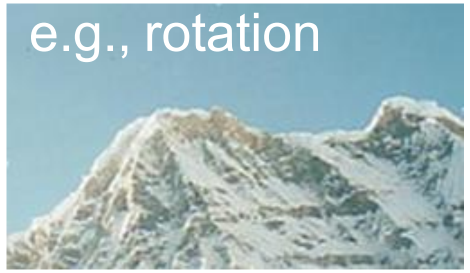
Goal: Repeatability

We want to detect (at least some of) the same points in both images.





Under geometric and photometric variations.



Kristen Grauman

Goal: Compactness and Efficiency

We want the representation to be as small and as fast as possible

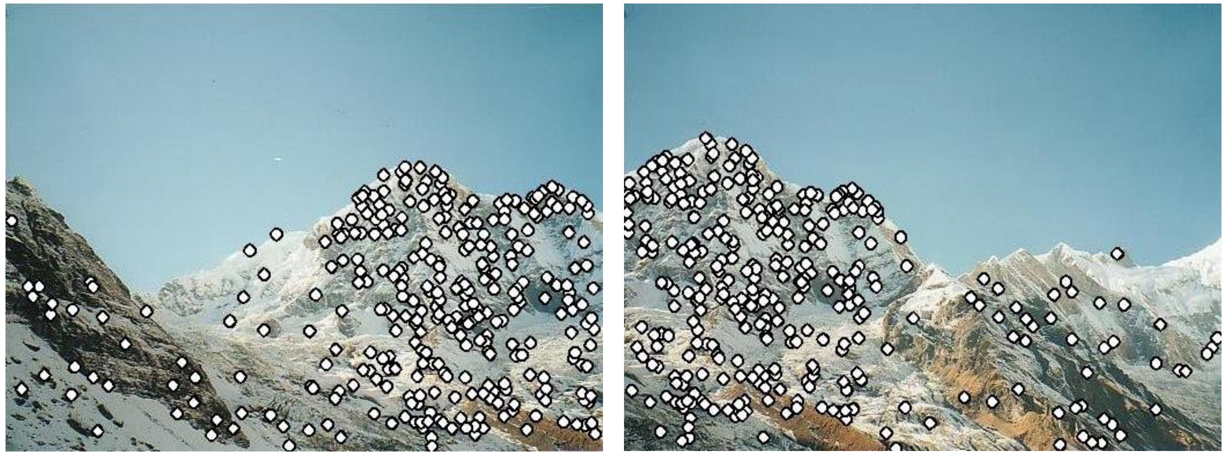
– Much smaller than a whole image

Sometimes, we'd like to run the detection procedure *independently* per image

- Match just the compact descriptors for speed.
- *Difficult!* We don't get to see 'the other image' at match time, e.g., object detection.

Kristen Grauman

Characteristics of good features



Distinctiveness

Each feature can be uniquely identified

Repeatability

The same feature can be found in several images despite differences:
- geometrically (translation, rotation, scale, perspective)
- photometrically (reflectance, illumination)

Compactness and efficiency

Many fewer features than image pixels; run independently per image

Kristen Grauman

Local features: main components

1) **Detection:**
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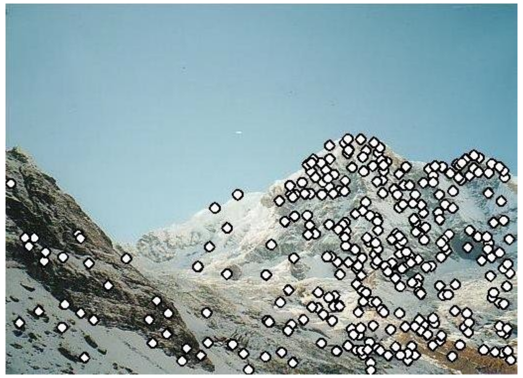
2) **Description:**
   Extract feature descriptor around each interest point as vector.

3) **Matching:**
   Compute distance between feature vectors to find correspondence.



Detection: Basic Idea

We do not know which other image locations the feature will end up being matched against …

But can compute how stable a location is in appearance with respect to small variations in its position

Something that "meaningfully stands out"!

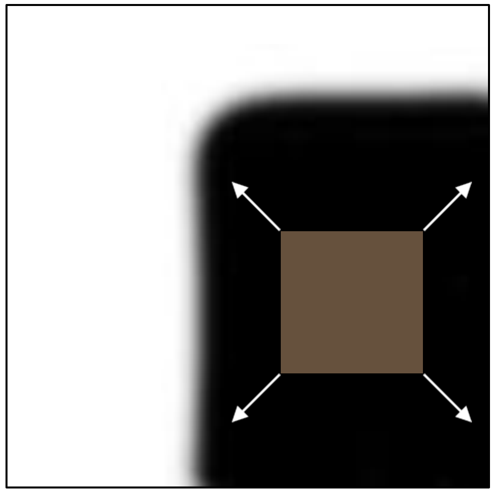
Strategy: Compare image patch against local neighbors

A. Efros

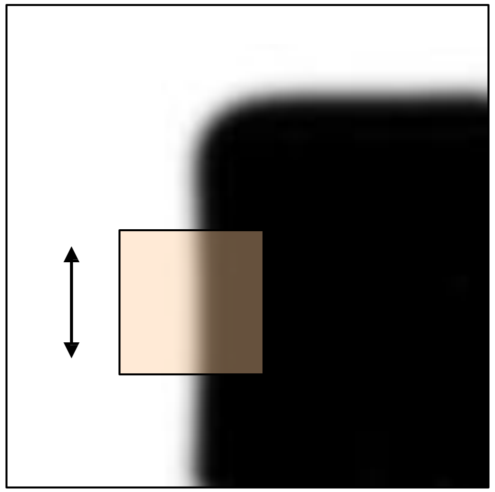
Detection: Basic Idea

Recognize corners by looking at small window.

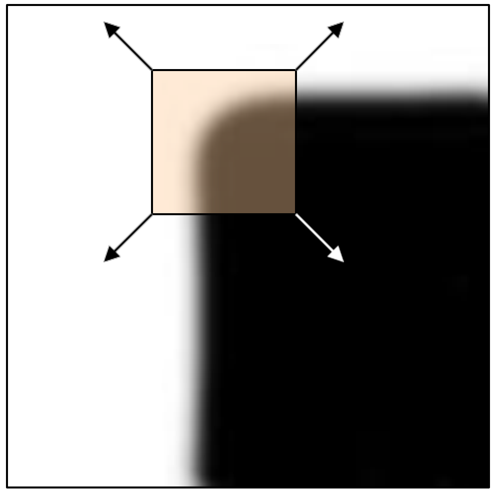
We want a window shift in *any direction* to give *a large change* in intensity.



"Flat" region: no change in all directions



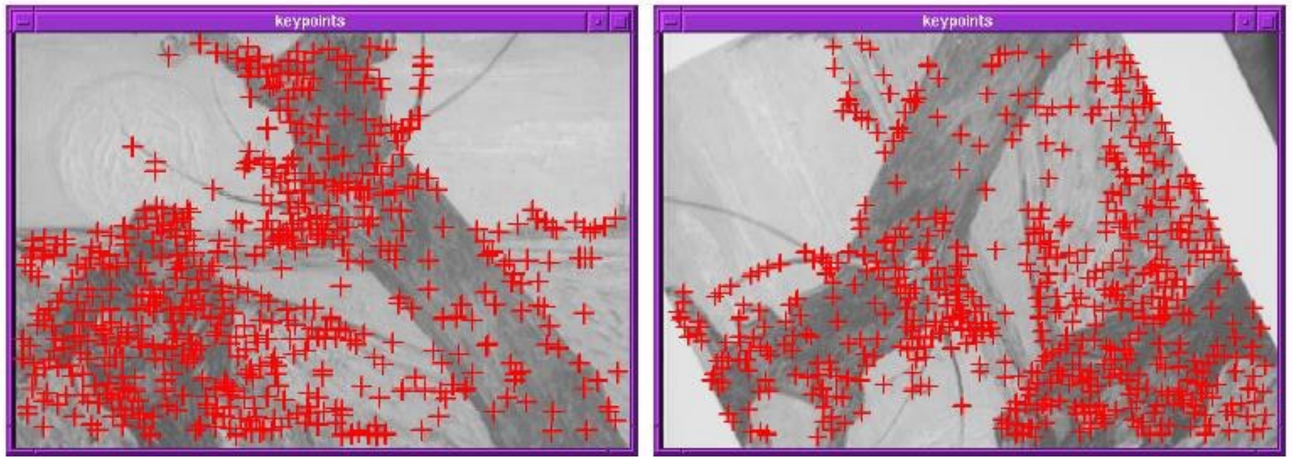
"Edge": no change along the edge direction



"Corner": significant change in all directions

A. Efros

Finding Corners



- Key property: in the region around a corner, image gradient has two or more dominant directions
- Corners are repeatable and distinctive

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ *Proceedings of the 4th Alvey Vision Conference*: pages 147--151.

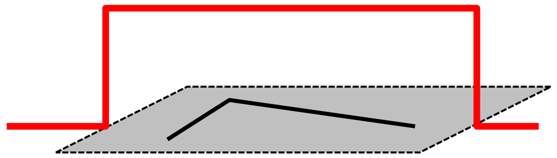
Corner Detection by Auto-correlation

Change in appearance of window *w*(*x*,*y*) for shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} \underbrace{w(x,y)}_{\text{Window function}} \left[\underbrace{I(x+u, y+v)}_{\text{Shifted intensity}} - \underbrace{I(x,y)}_{\text{Intensity}} \right]^2$$

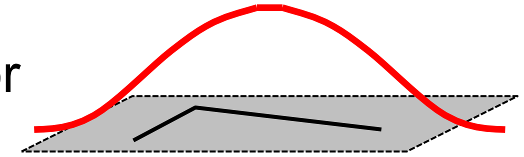
Also called 'sum of squared differences'

Window function $w(x,y) =$



1 in window, 0 outside

or



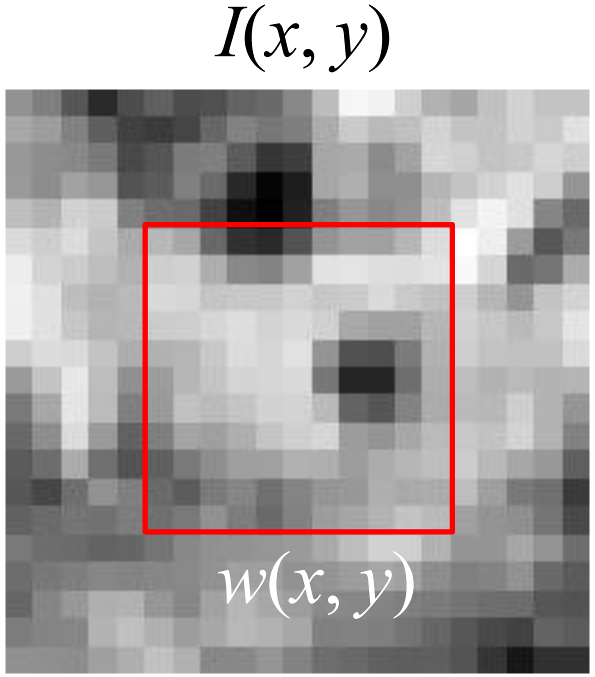
Gaussian

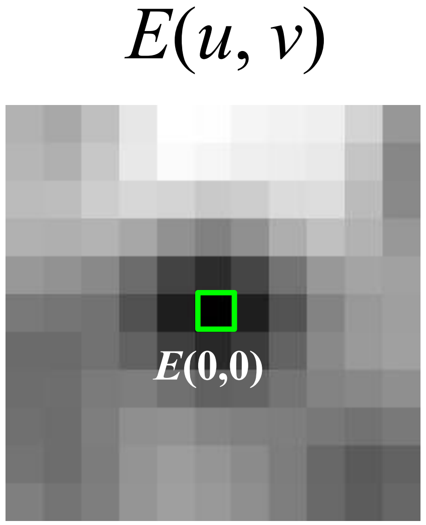
Source: R. Szeliski

Corner Detection by Auto-correlation

Change in appearance of window *w*(*x*,*y*) for shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

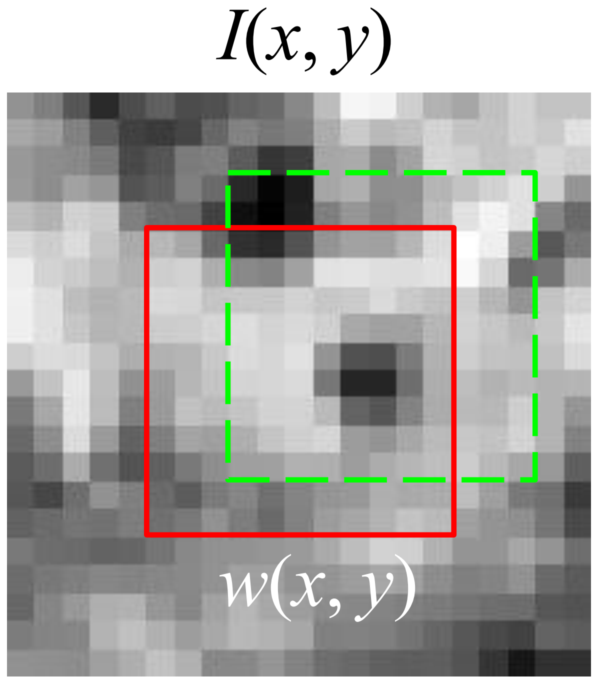


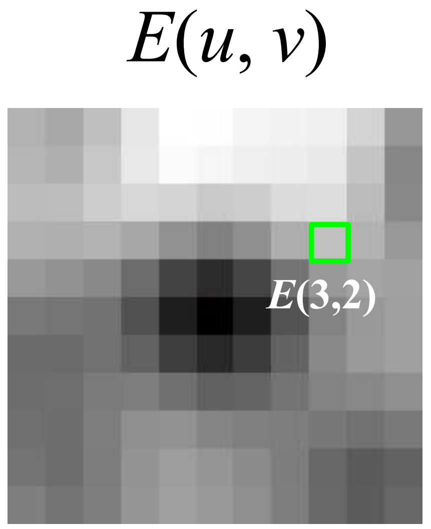


Corner Detection by Auto-correlation

Change in appearance of window $w(x,y)$ for shift $[u,v]$:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$





Corner Detection by Auto-correlation

Change in appearance of window *w*(*x*,*y*) for shift [*u*,*v*]:

$$E(u,v) = \sum_{x,y} w(x,y) \left[I(x+u, y+v) - I(x,y) \right]^2$$

We want to discover how E behaves for small shifts (corner = function value change fast w.r.t small shifts)

But this is very slow to compute naively.
O(window_width^2 * shift_range^2 * image_width^2)

O(11^2 * 11^2 * 600^2) = 5.2 billion of these 14.6k ops per image pixel

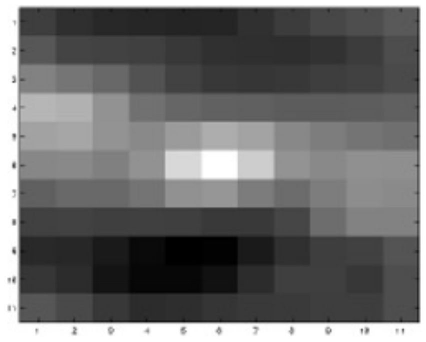
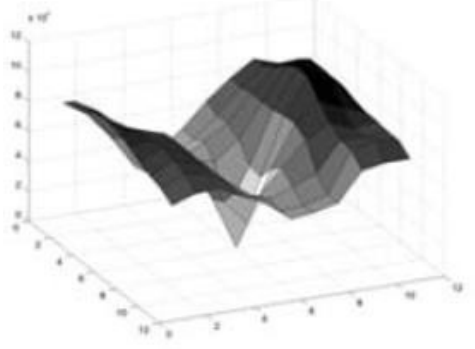
Corner Detection by Auto-correlation

Change in appearance of window *w*(*x*,*y*) for shift [*u*,*v*]:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

….But we know the response in *E* that we are looking for – **strong peak!**

- *E needs to “change” fast w.r.t. u & v*
- *(from u = 0, v = 0)*

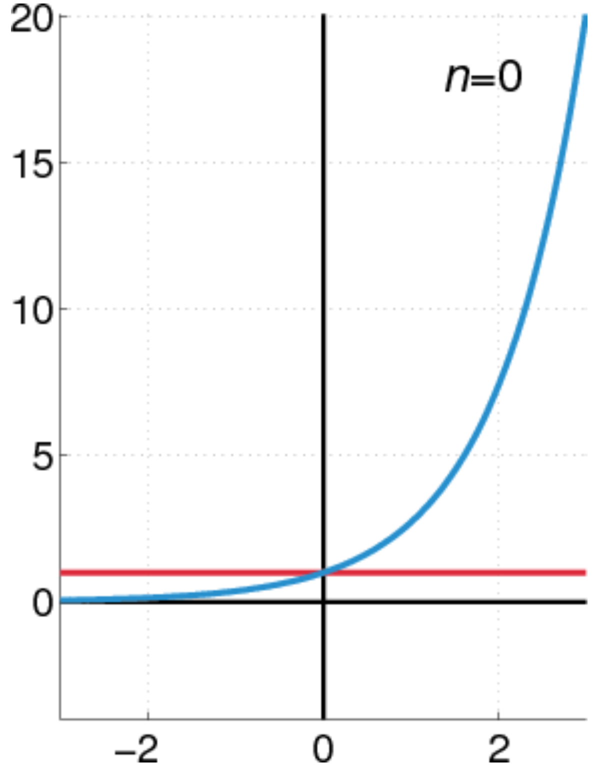
Recall: Taylor series expansion

A function f can be represented by an infinite series of its derivatives at a single point *a*:

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots.$$

Wikipedia

As we care about window centered, we set *a = 0* (MacLaurin series)



Approximation of $f(x) = e^x$ centered at f(0)

Corner Detection: Mathematics (Simplified)

- First-order Taylor approximation for small shifts (u, v):

$$I(x+u, y+v) \approx I(x, y) + I_x u + I_y v$$

(Why first-order is good enough?)

- Let's plug this into $E(u, v)$:

$$E(u, v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x, y)]^2$$

(We ignore W here for simplicity)

$$\approx \sum_{(x,y)\in W} \left[I(x, y) + I_x u + I_y v - I(x, y)\right]^2$$

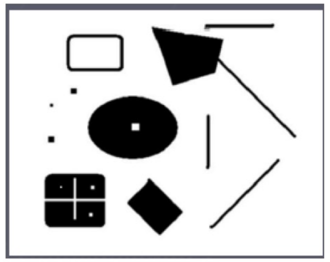
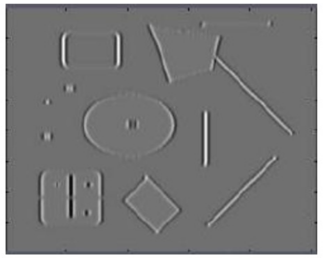
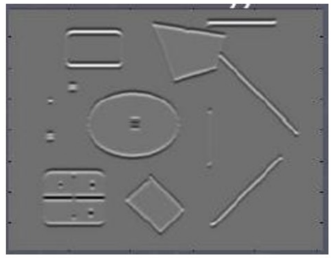
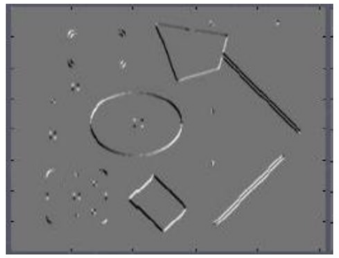
$$= \sum_{(x,y)\in W} \left[I_x u + I_y v\right]^2 = \sum_{(x,y)\in W} I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2$$

Corners as distinctive interest points

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives
(averaged in neighborhood of a point)

Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

James Hays

Corners as distinctive interest points

$$E(u,v) \approx [u \ \ v] \ M \begin{bmatrix} u \\ v \end{bmatrix}$$

$$M = \sum w(x,y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives
(averaged in neighborhood of a point)

Reminder/Refresher:

- Our goal is to find ***(x,y)*** likely at corner. ***(u,v)*** denotes a small neighborhood near ***(x,y)***
- ***E(u,v)*** is evaluated at each ***(x,y).*** Its "parameter" depends on ***(x,y),*** e.g., ***M***
- For each ***(x,y)***, we want to find "extreme" values for ***E(u,v)*** -- *now reducing to analyzing* ***M***
- *M* encodes the **"variation" level** of ***E(u,v)*** in the small ***(u,v)*** neighborhood – how to decode?

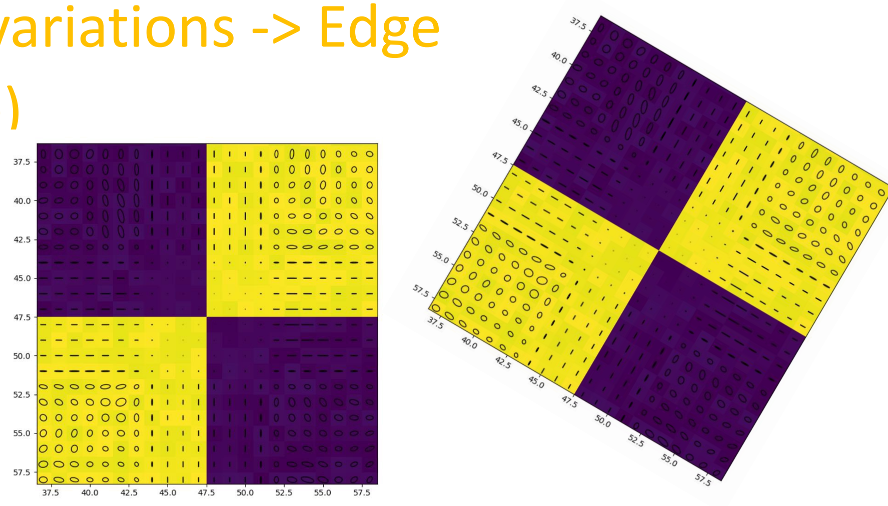
James Hays

Let’s go back to our goal: corner detection

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- For detecting “cornerness”:
  - Do we care about the change orientation? No
  - Do we care about the change “steepness”? Yes, that is “all we need”

- So, looking at the M approximation now, what we really want?
  - **What if I^2_x, I^2_y, $I_x I_y$ are all small?** No variations -> flat area
  - **What if only I^2_x is large?** Only x-direction has large variations -> Edge
  - **How about only large I^2_y, or $I_x I_y$?** Same thing (edge)
  - **Then, how about letting I^2_x, I^2_y, $I_x I_y$ all be large?**
    - Sufficient, but not necessary…
    - **The missing key: Rotation Invariance**



Eigenvalue Analysis (your old friend: PCA)

- **Goal:** Describe the "overall intensity variations" in the window, *regardless of rotation*!

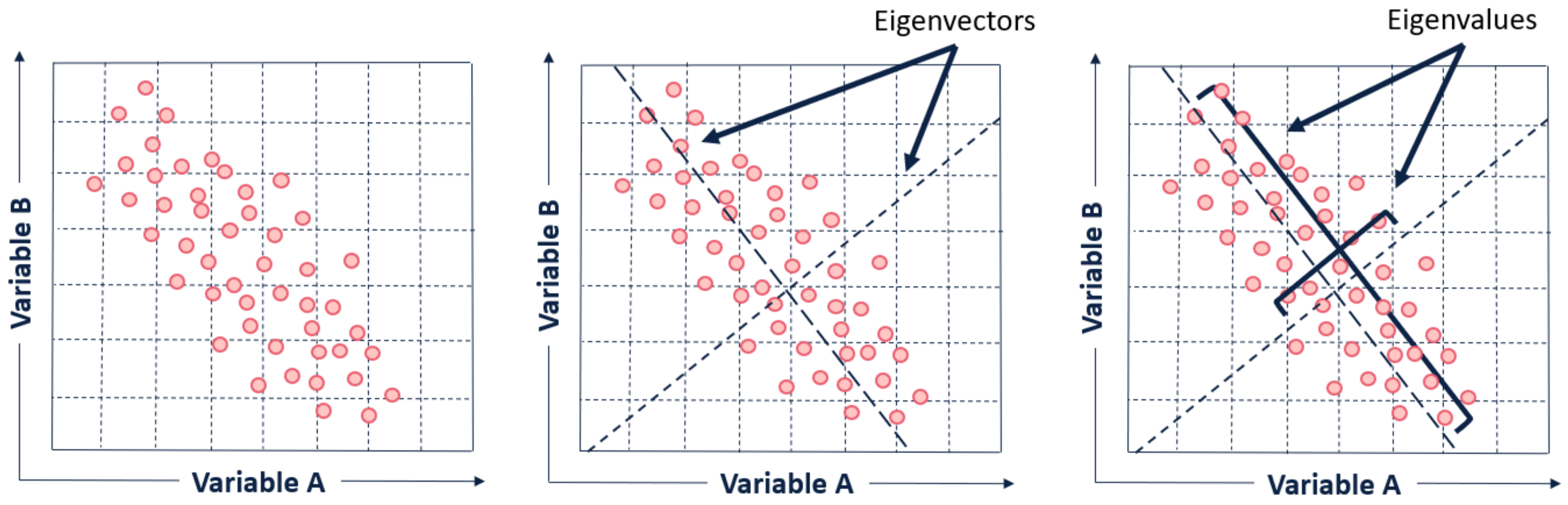
… by **eigenvalue analysis**

$$E(u,v) \cong [u,v] \quad M \begin{bmatrix} u \\ v \end{bmatrix}$$

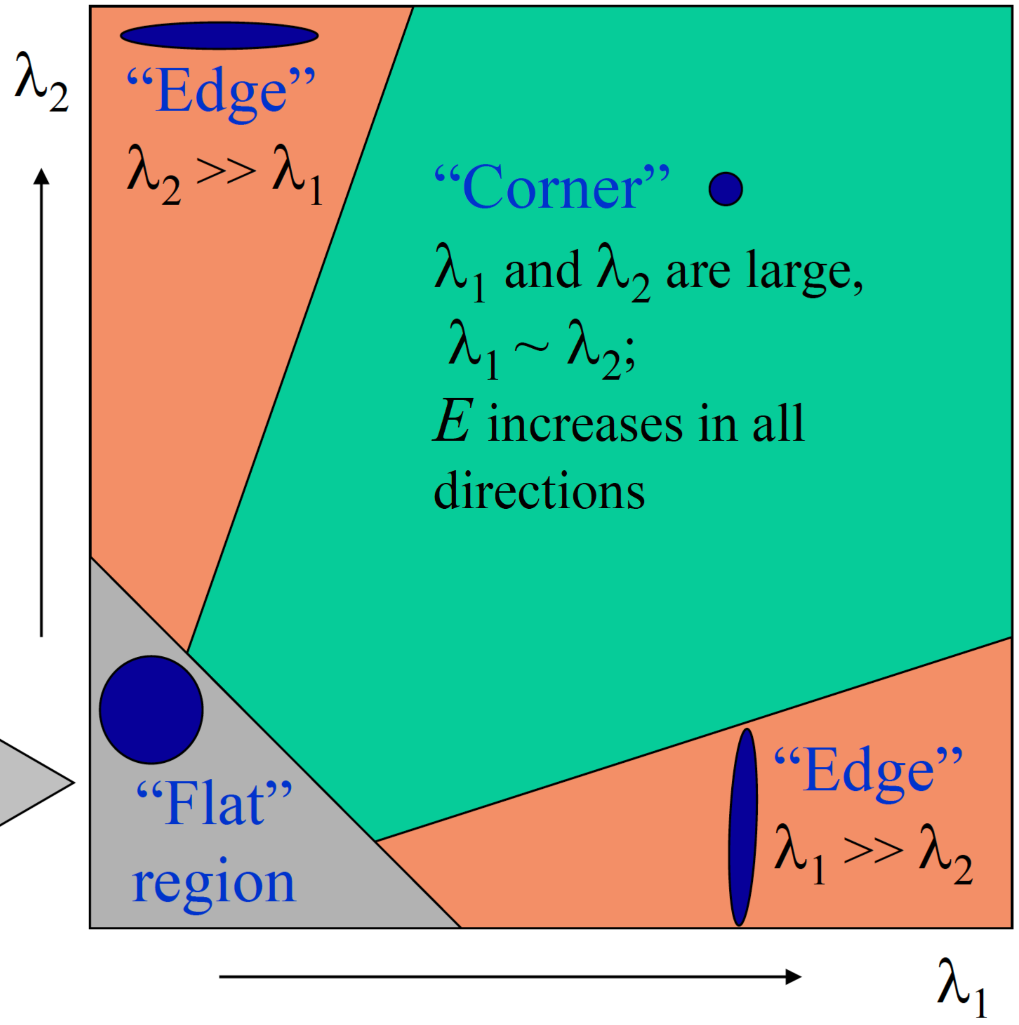
λ_1, λ_2 – eigenvalues of M

What PCA can tell us about the overall "variations"

- *Eigenvectors told us the 1st/2nd/3rd … major directions of change*
- *Correspondingly, eigenvalues capture "change rate" along each of those directions*


Variable B
Variable A
Eigenvectors
Variable B
Variable A
Eigenvalues
Variable B
Variable A

Categorizing image points using *M* eigenvalues



λ_1 and λ_2 are small;
E is almost constant in all directions

Categorizing image points using *M* eigenvalues

Cornerness score:

$$C = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: some small constant (~0.04 to 0.06)

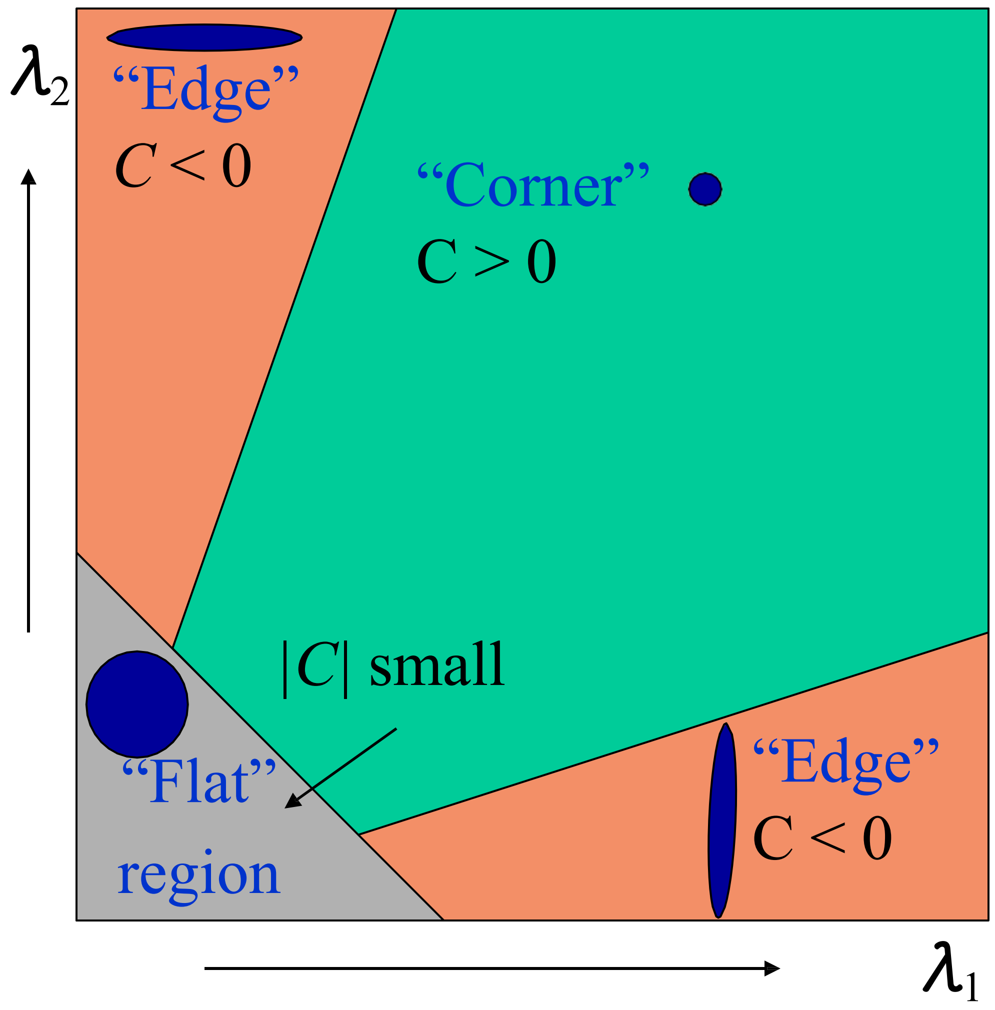
To see why:

Assume $\lambda_1 = k\lambda_2$,

C = [k - α (k+1)^2] $\lambda_2{}^2$

Then analyze: k - α (k+1)^2

What if k is very large? very small? around 1?



Categorizing image points using *M* eigenvalues

Cornerness score:

$$C = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: some constant (~0.04 to 0.06)

Remember your linear algebra:

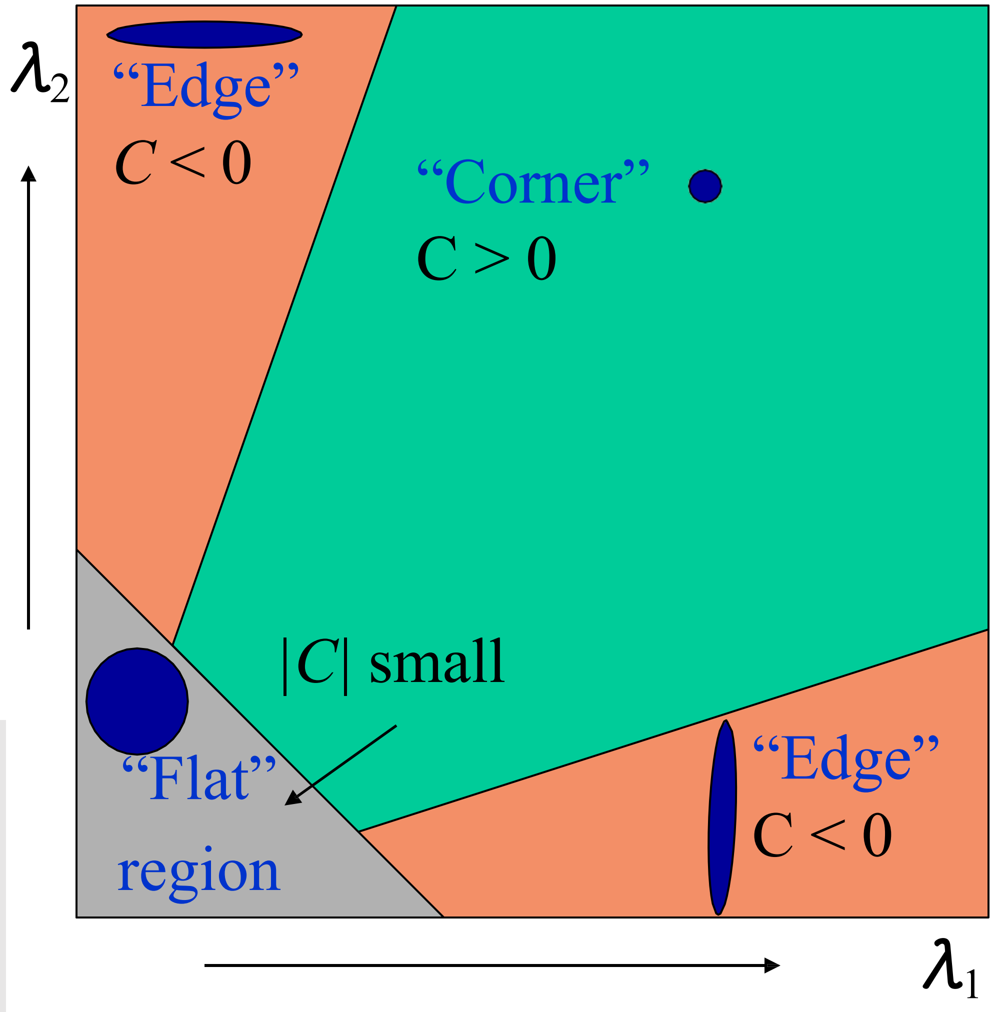
Determinant: $\det(A) = \prod_{i=1}^{n} \lambda_i = \lambda_1 \lambda_2 \cdots \lambda_n .$
(diagonal matrices)

Trace: $\operatorname{tr}(A) = \sum_i \lambda_i .$

$$C = \det(M) - \alpha \operatorname{Tr}^2(M)$$

Avoids explicit eigenvalue computation!

(why?)



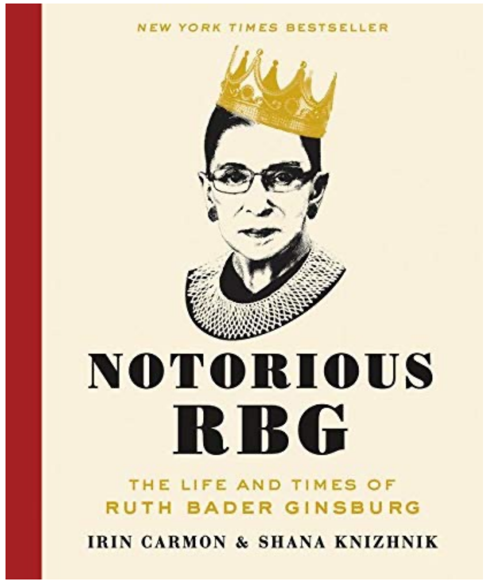
This is the "notorious" Harris corner detector!

1) Compute *M* matrix for each window to recover the *cornerness* score C.

   Note: We can build *M* purely from the per-pixel image derivatives!

2) Threshold to find pixels which give large corner response (C > threshold).

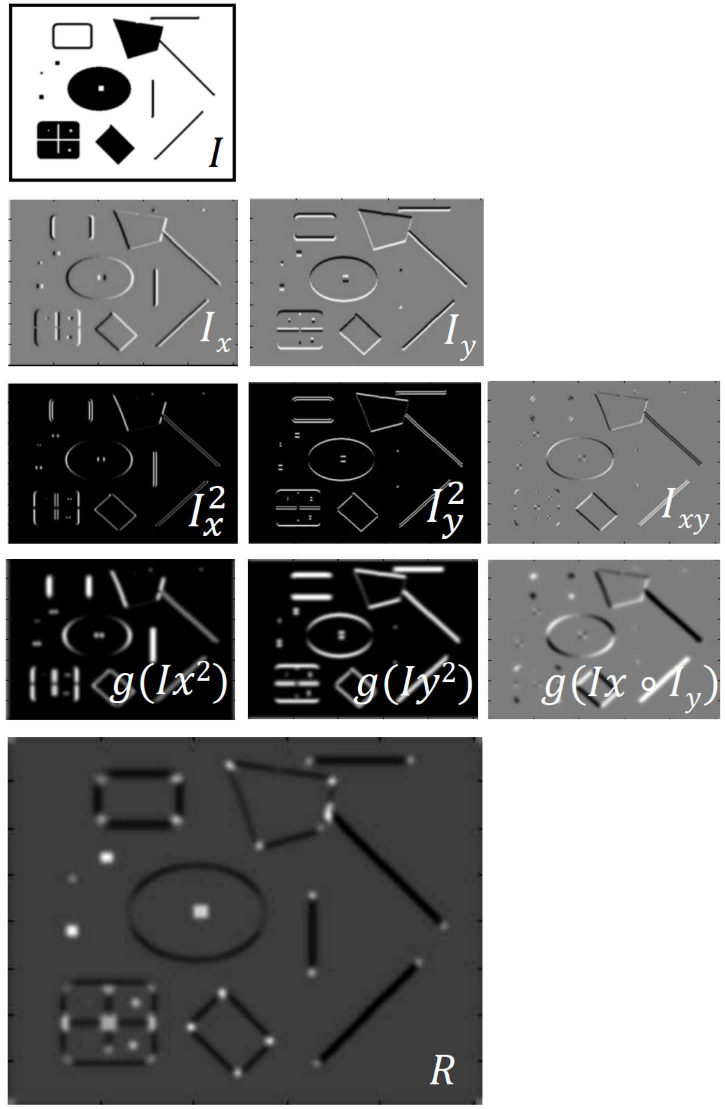
3) Find the local maxima pixels, i.e., non-maximal suppression.


NEW YORK TIMES BESTSELLER
NOTORIOUS RBG
THE LIFE AND TIMES OF RUTH BADER GINSBURG
IRIN CARMON & SHANA KNIZHNIK

C.Harris and M.Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Harris Corner Detector [Harris88]

James Hays



0. Input image
   We want to compute M at each pixel.

1. Compute image derivatives (optionally, blur first).

2. Compute M components
   as squares of derivatives.

3. Gaussian filter *g()* with width σ

$$= g(I_x^2),\ g(I_y^2),\ g(I_x \circ I_y)$$

Reminder: $a \circ b$ is Hadamard product (element-wise multiplication)

4. Compute cornerness

$$C = \det(M) - \alpha \operatorname{trace}(M)^2$$
$$= g(I_x^2) \circ g(I_y^2) - g(I_x \circ I_y)^2$$
$$-\alpha\left[g(I_x^2) + g(I_y^2)\right]^2$$

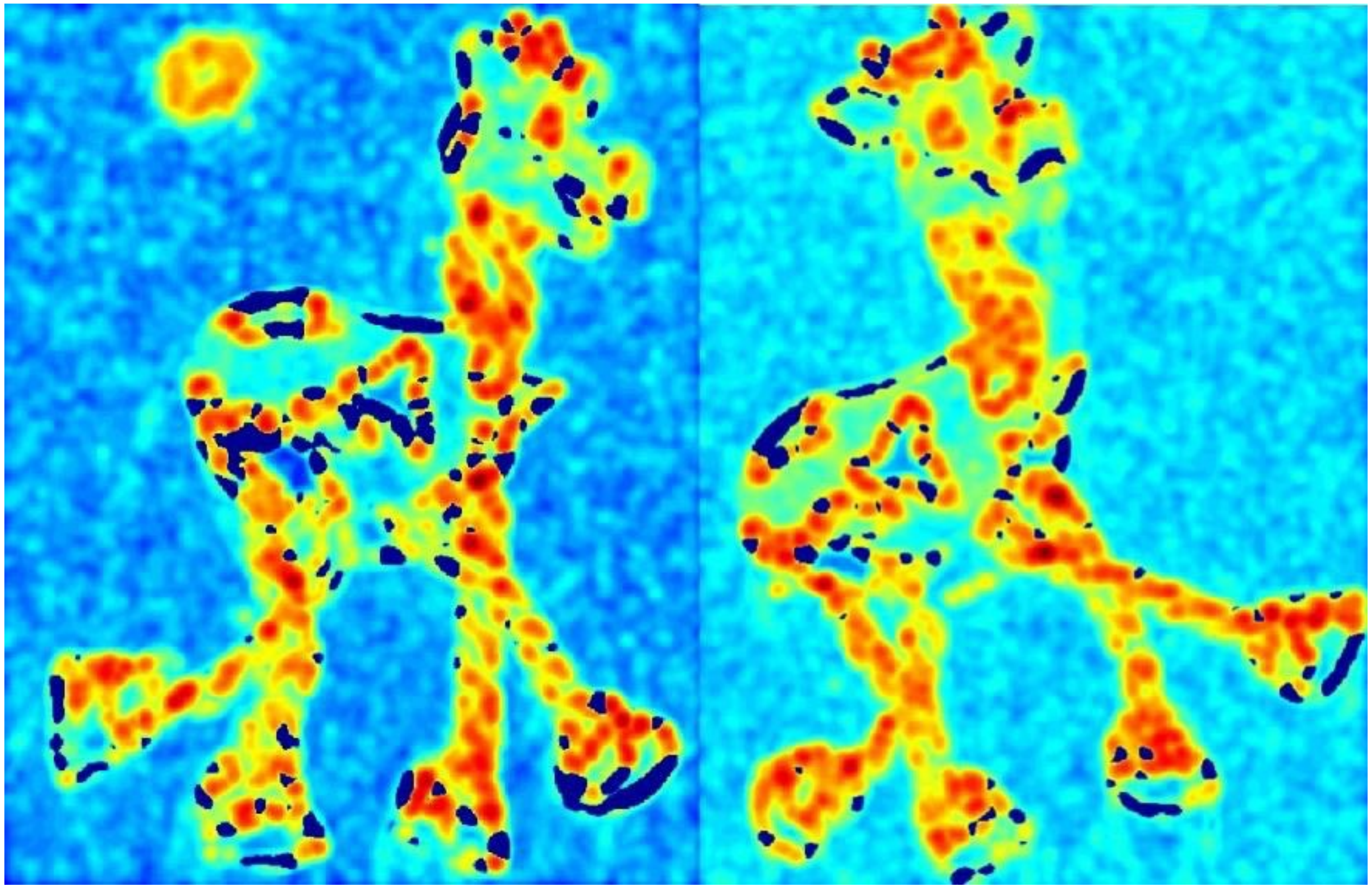
5. Threshold on C to pick high cornerness

6. Non-maximal suppression to pick peaks.

Harris Detector: Steps

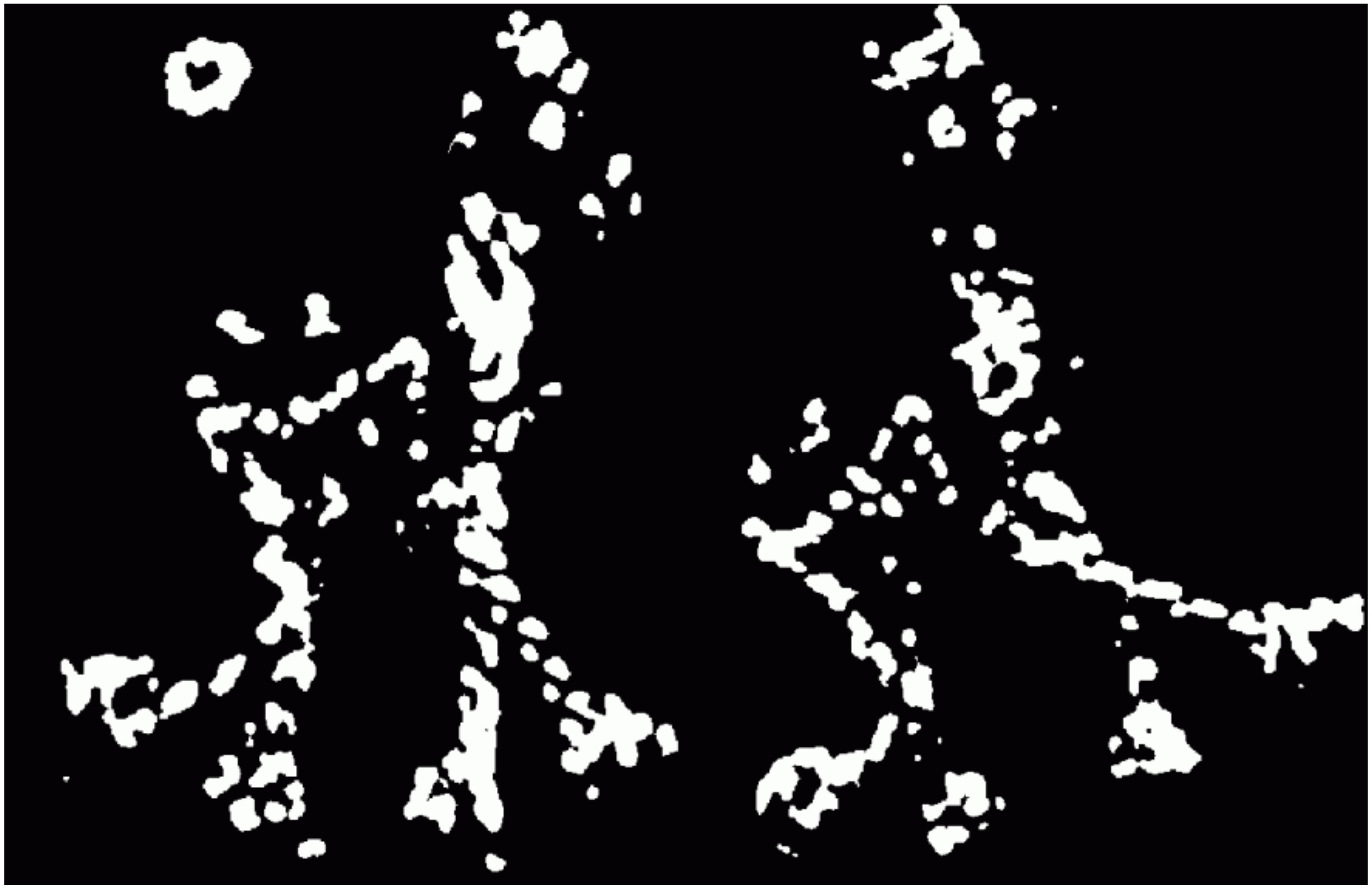
Harris Detector: Steps

Compute corner response C



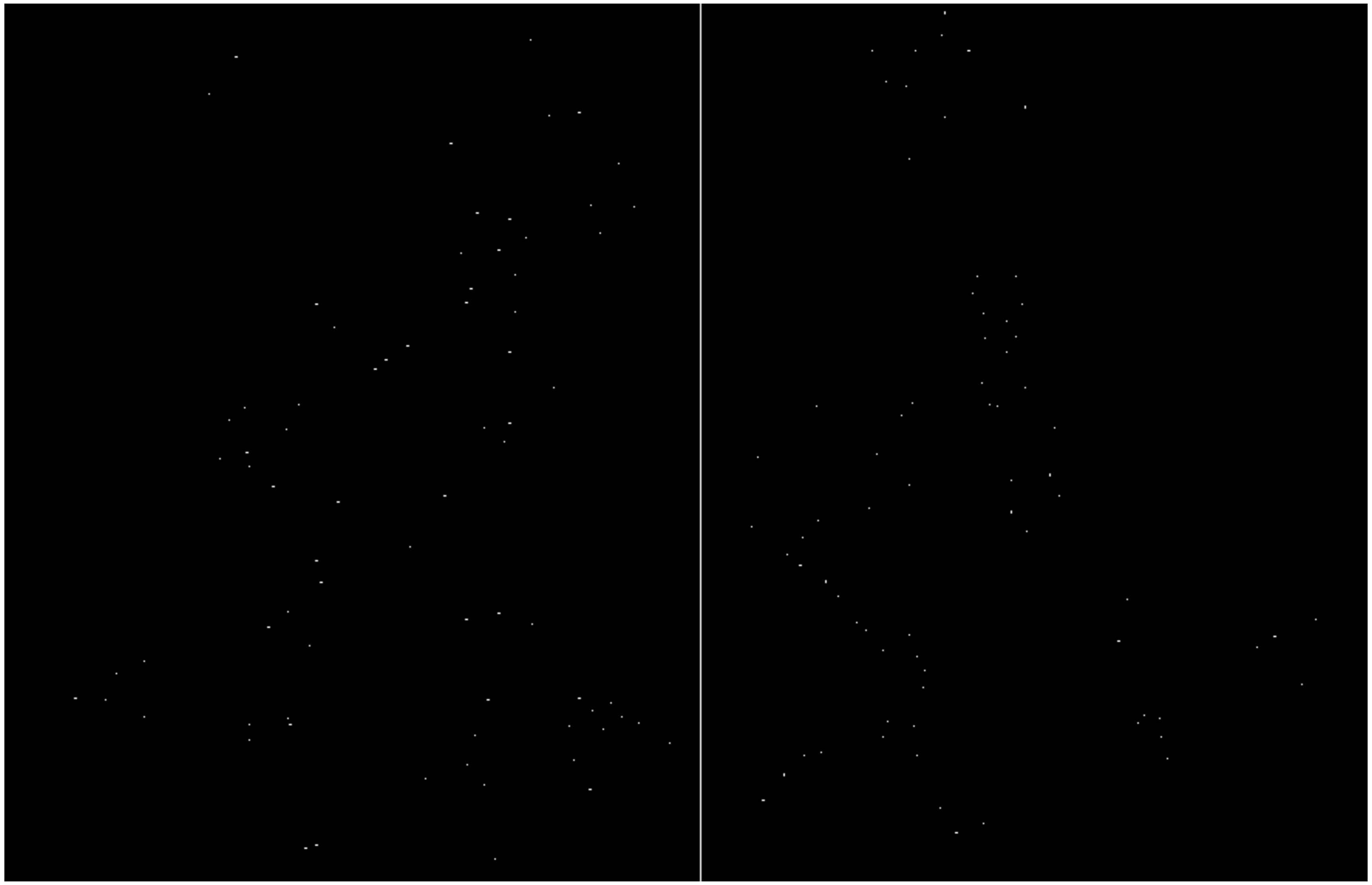
Harris Detector: Steps

Find points with large corner response: C > threshold

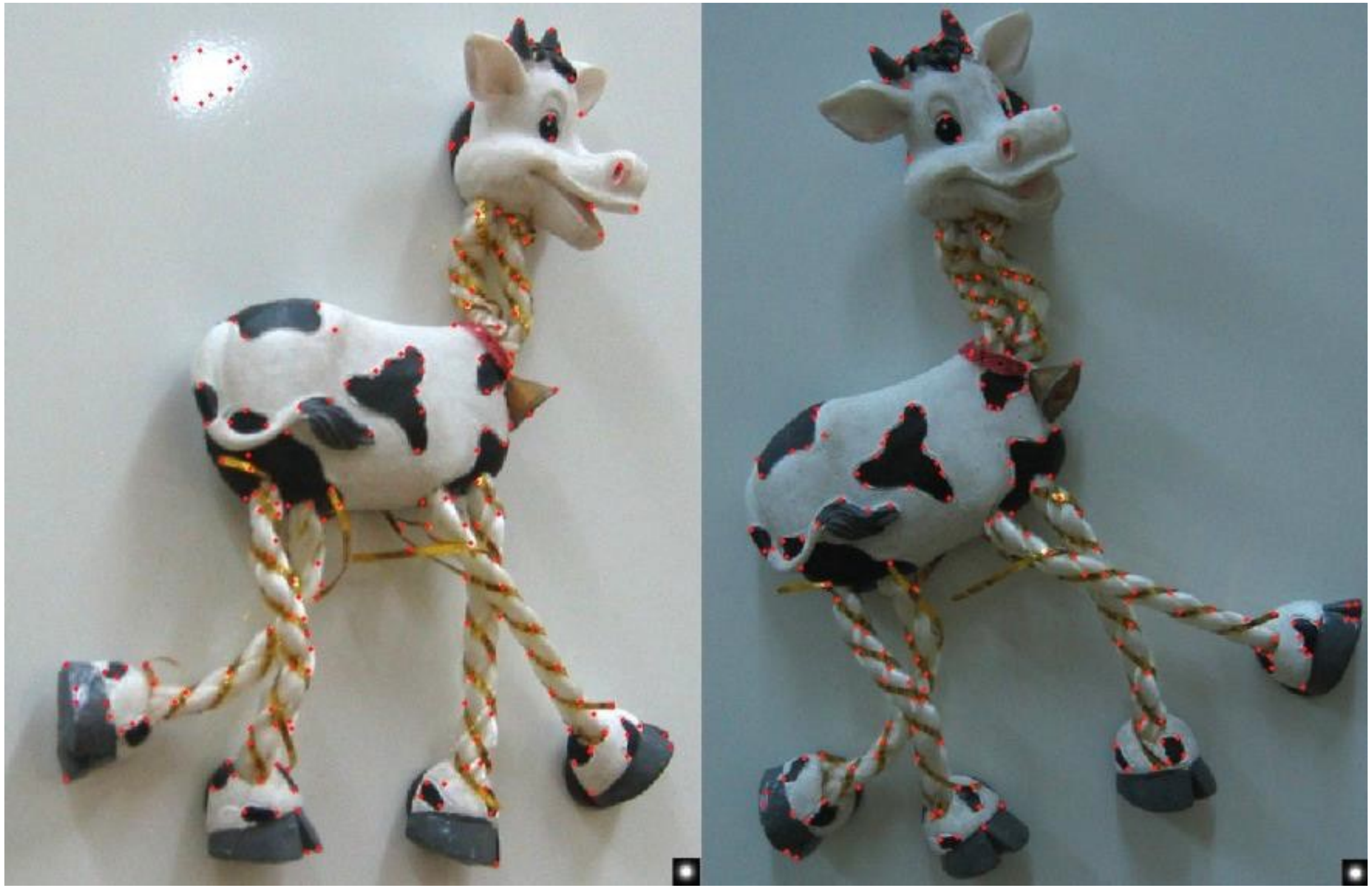


Harris Detector: Steps

Take only the points of local maxima of C

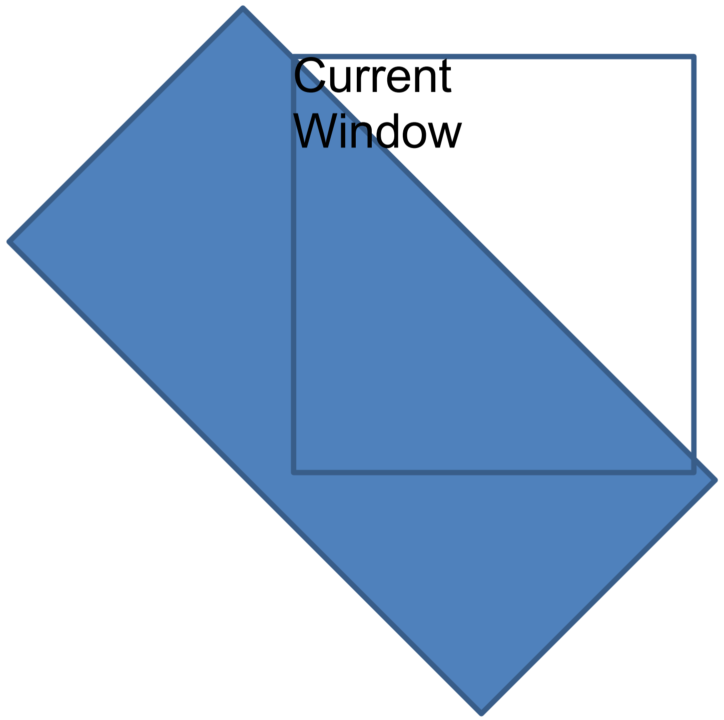


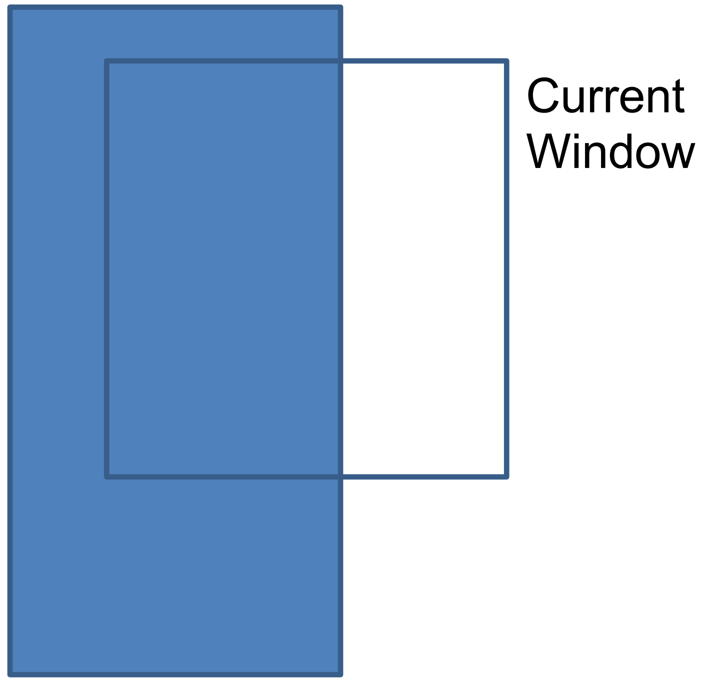
Harris Detector: Steps



Harris Corners – Why so complicated?

- Can’t we just check for regions with lots of gradients in the x and y directions (or any specific)?
  - No! A diagonal line or alike would satisfy that criteria

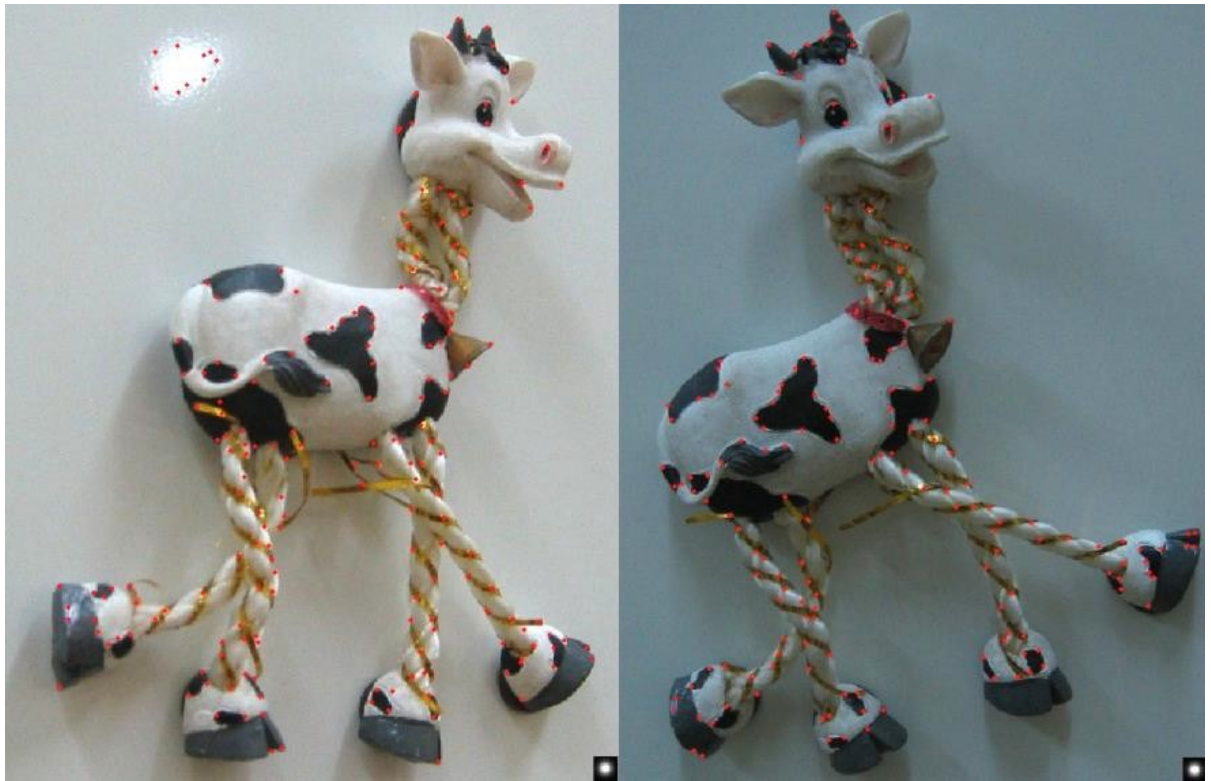




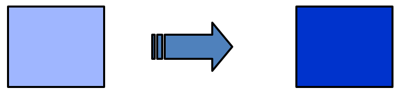
Invariance and covariance

Are locations *invariant* to photometric transformations and *covariant* to geometric transformations?

- **Invariance:** image is transformed and corner locations do not change
- **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations



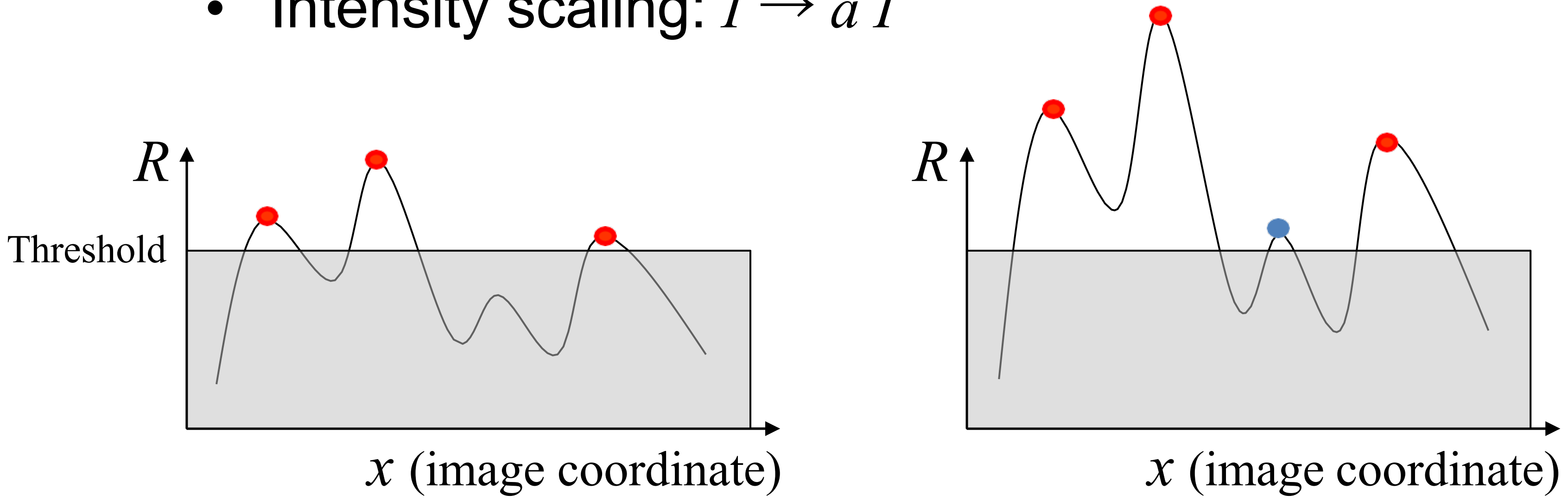
Affine intensity change



$$I \rightarrow a\,I + b$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

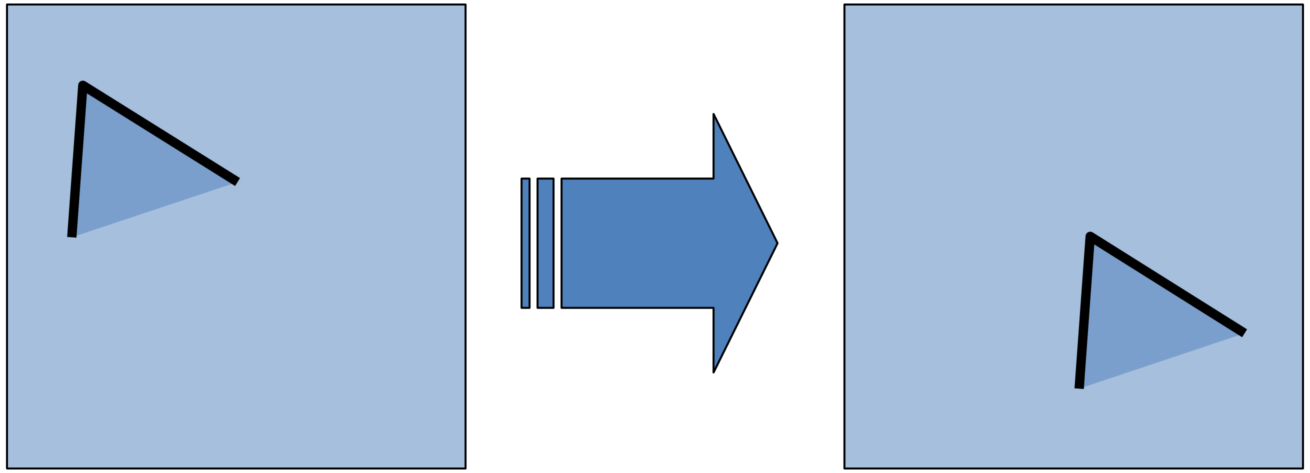
- Intensity scaling: $I \rightarrow a\,I$



Partially invariant to affine intensity change

James Hays

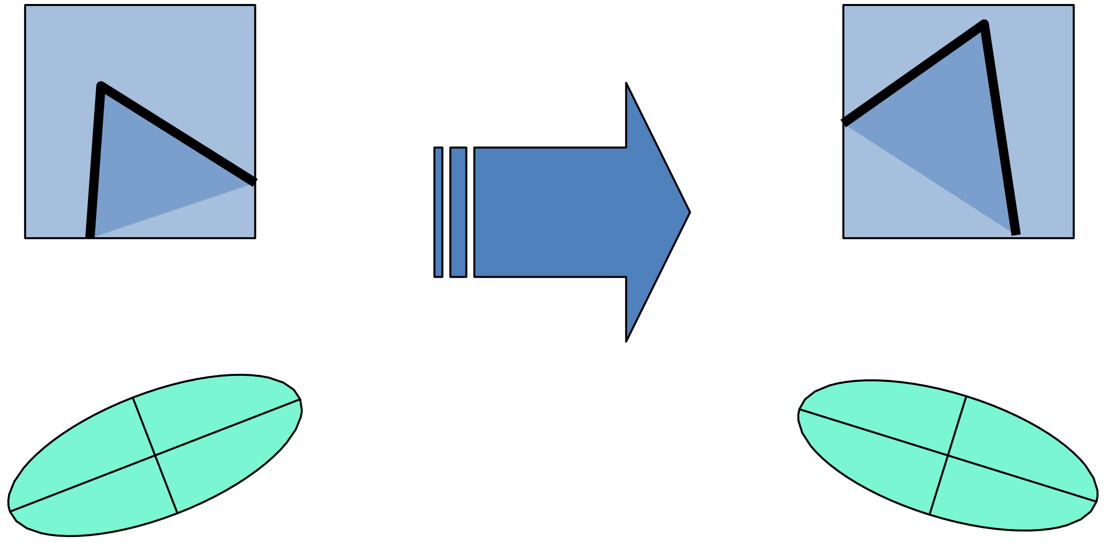
Image translation



- Derivatives and window function are shift-invariant.

Corner location is covariant w.r.t. translation

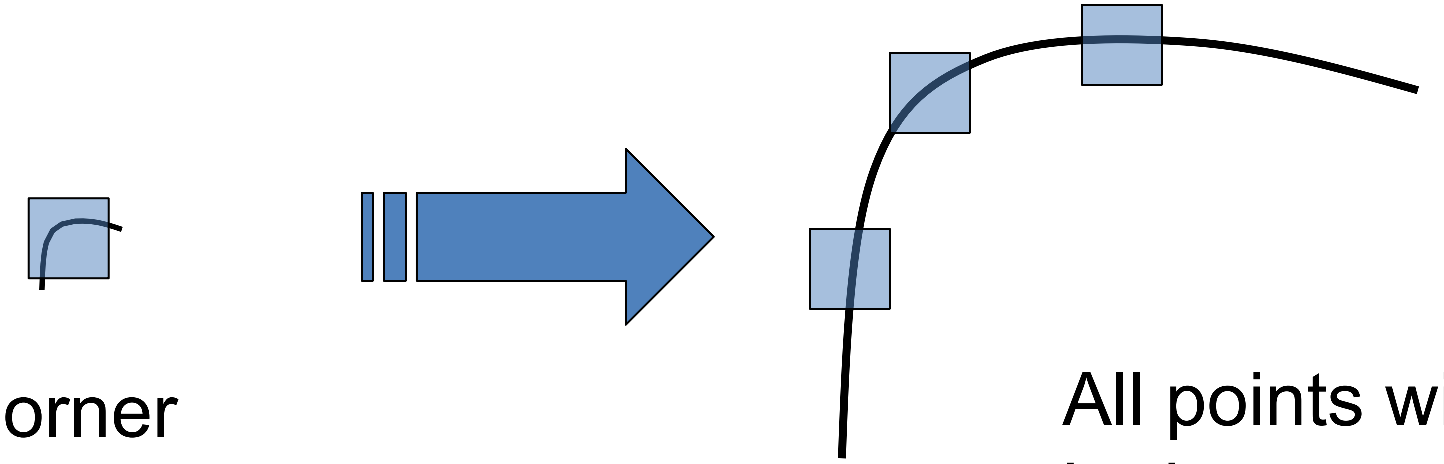
Image rotation



Second moment ellipse rotates but its shape (i.e., eigenvalues) remains the same.

Corner location is covariant w.r.t. rotation

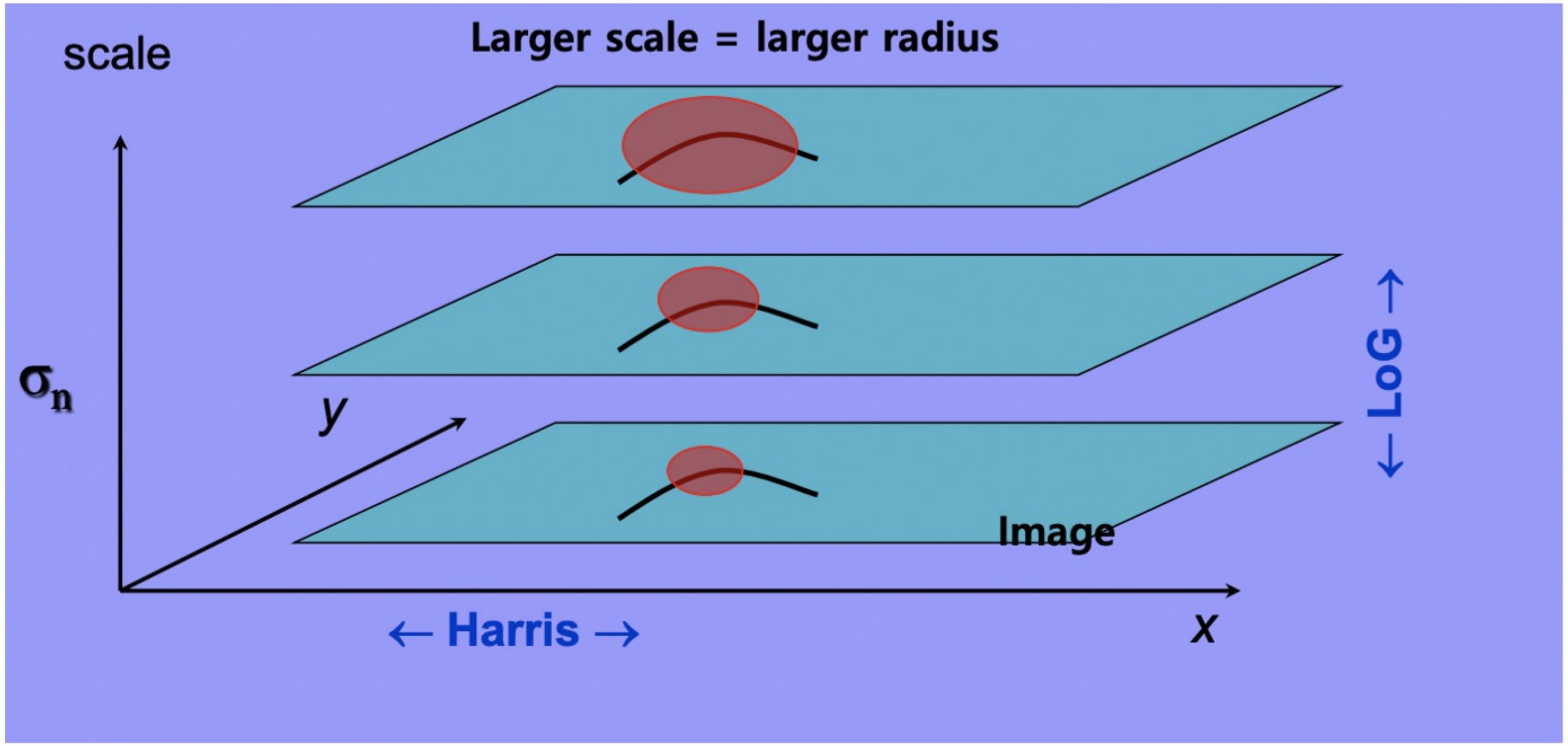
Scaling



All points will be incorrectly classified as edges

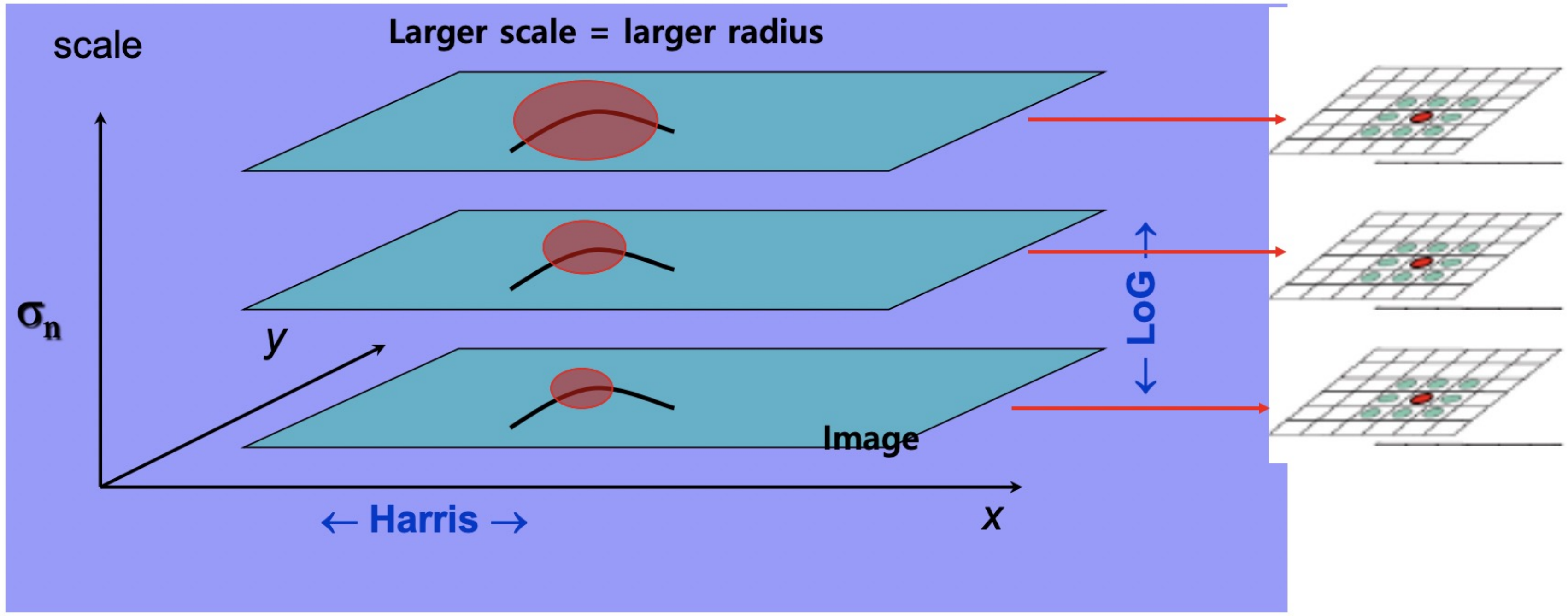
Corner location is not covariant to scaling!

Harris-Laplace detector [Mikolajczyk '01]



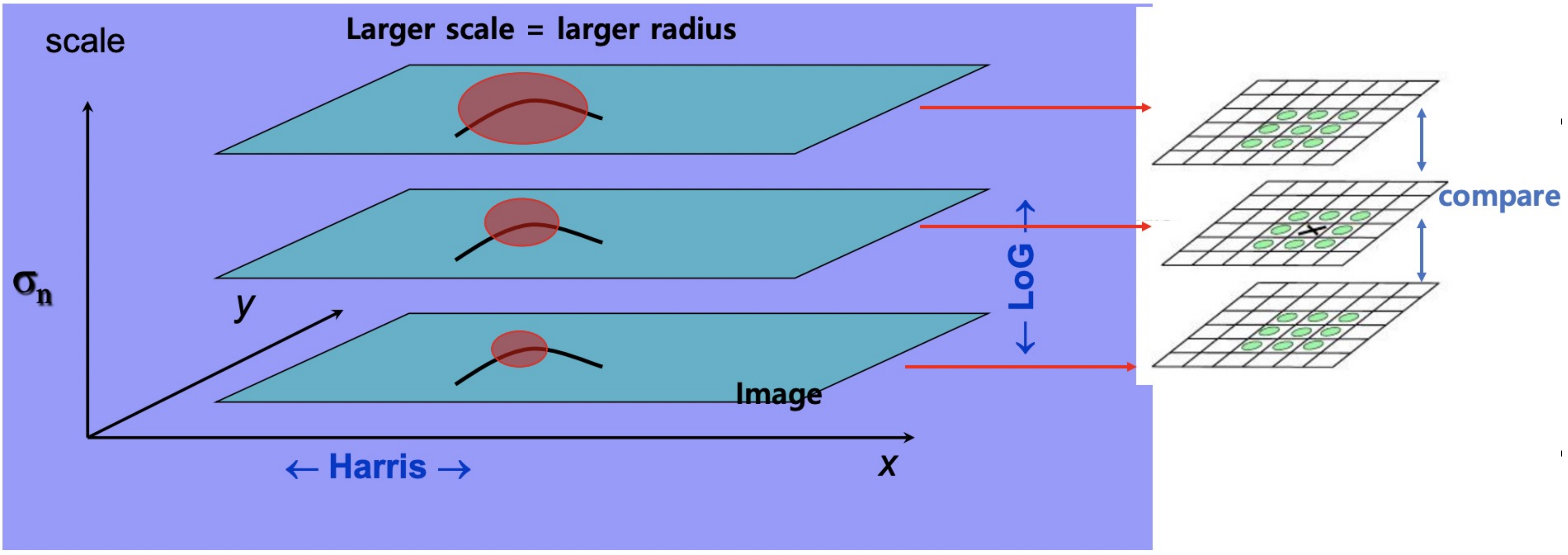
- **Step 1.** Build the Laplacian Pyramid of one image

Harris-Laplace detector [Mikolajczyk '01]



- **Step 1.** Build the Laplacian Pyramid of one image
- **Step 2**. Run the *Harris* detector to compute interest points *at each scale*

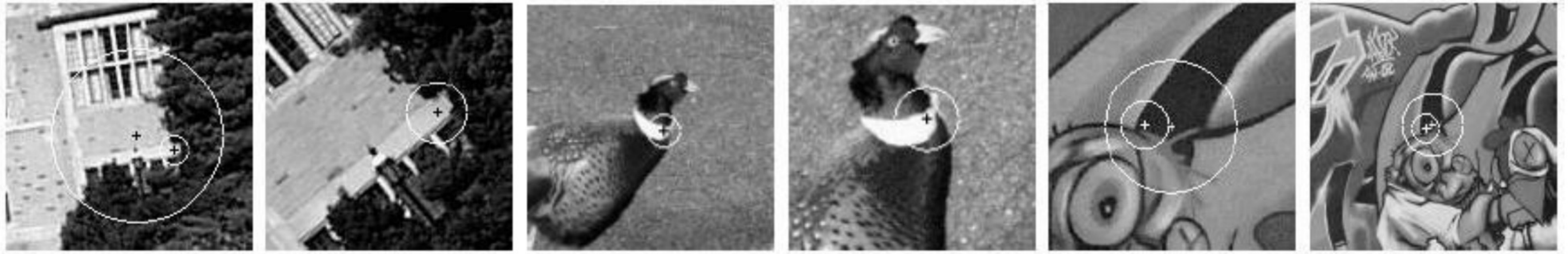
Harris-Laplace detector [Mikolajczyk '01]



- **Step 1.** Build the Laplacian Pyramid of one image
- **Step 2**. Run the *Harris* detector to compute interest points *at each scale*
- **Step 3**. Non-maximal suppression, not only at each scale, **but also at adjacent scales**

Harris-Laplace detector [Mikolajczyk '01]

- **A scale-invariant detector!**
  - *Automatically search for the right scale to detect corners, by "multi-scaling then max-pooling"*



Harris-Laplace points

A Longer List of Local Keypoint Detectors…

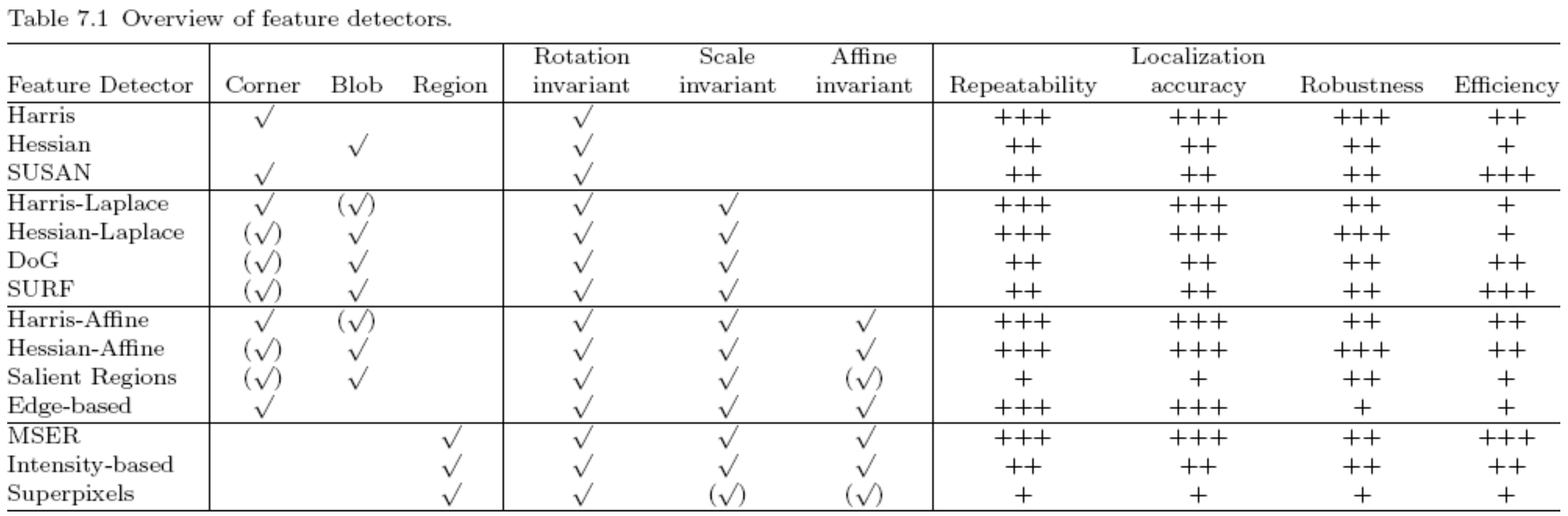


Table 7.1 Overview of feature detectors.

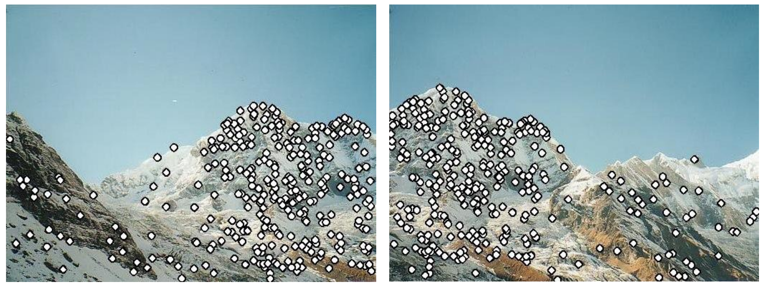
Feature Detector	Corner	Blob	Region	Rotation invariant	Scale invariant	Affine invariant	Repeatability	Localization accuracy	Robustness	Efficiency
Harris	✓			✓			+++	+++	+++	++
Hessian		✓		✓			++	++	++	+
SUSAN	✓			✓			++	++	++	+++
Harris-Laplace	✓	(✓)		✓	✓		+++	+++	++	+
Hessian-Laplace	(✓)	✓		✓	✓		+++	+++	+++	+
DoG	(✓)	✓		✓	✓		++	++	++	++
SURF	(✓)	✓		✓	✓		++	++	++	+++
Harris-Affine	✓	(✓)		✓	✓	✓	+++	+++	++	++
Hessian-Affine	(✓)	✓		✓	✓	✓	+++	+++	+++	++
Salient Regions	(✓)	✓		✓	✓	(✓)	+	+	++	+
Edge-based	✓			✓	✓	✓	+++	+++	+	+
MSER			✓	✓	✓	✓	+++	+++	++	+++
Intensity-based			✓	✓	✓	✓	++	++	++	++
Superpixels			✓	✓	(✓)	(✓)	+	+	+	+

- It is always of interest to collect more points with more detectors, for more possible matches
- Need consider location preciseness, variation robustness, and flexibility in region shapes
- Best choice often application dependent
  - Harris/Harris-Laplace work well for many natural image categories
  - MSER works well for buildings and printed things
  - Although no "silver bullet", all detectors/descriptors shown here work well in general

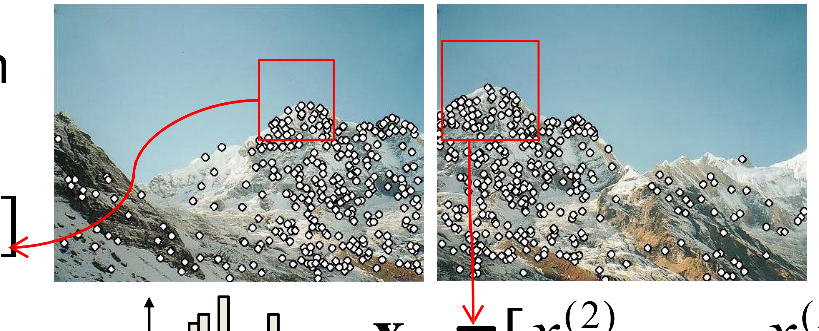
Tuytelaars Mikolajczyk 2008

Local features: main components

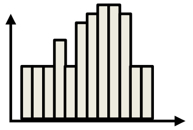
1) **Detection:**
Find a set of distinctive key points.



2) **Description:**
Extract feature descriptor around each interest point as vector.

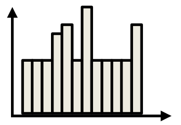


x_1



$$\mathbf{x}_1 = [x_1^{(1)}, \dots, x_d^{(1)}]$$

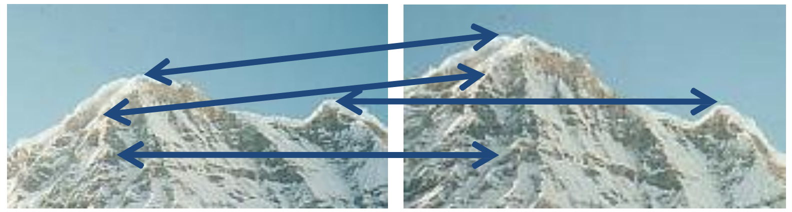
x_2



$$\mathbf{x}_2 = [x_1^{(2)}, \dots, x_d^{(2)}]$$

3) **Matching:**
Compute distance between feature vectors to find correspondence.

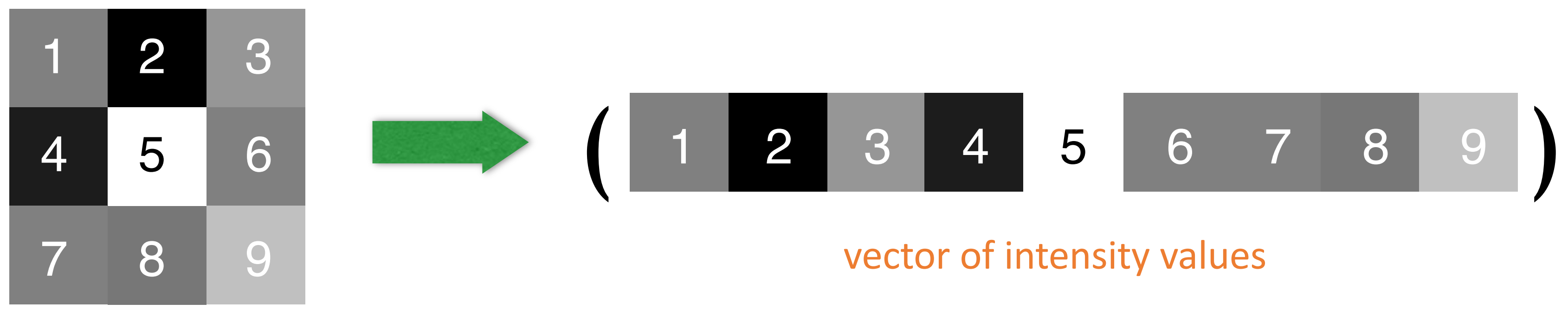
$$d(\mathrm{x}_1, \mathrm{x}_2) < T$$



K. Grauman, B. Leibe

Image patch

Just use the pixel values of the patch



vector of intensity values

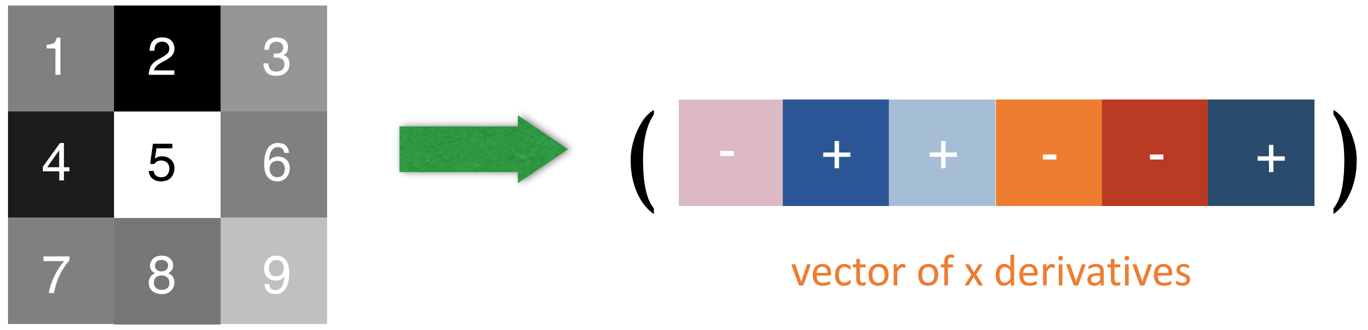
Perfectly fine if geometry and appearance is unchanged (a.k.a. template matching)

What are the problems?

How can you be less sensitive to absolute intensity values?

Image gradients

Use pixel differences

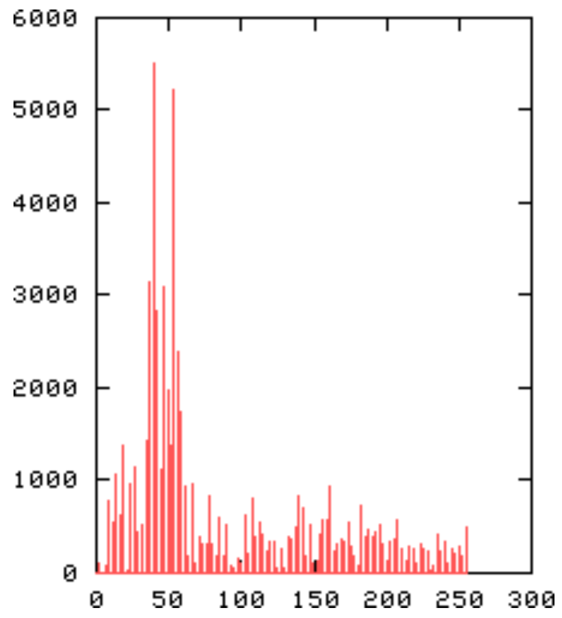


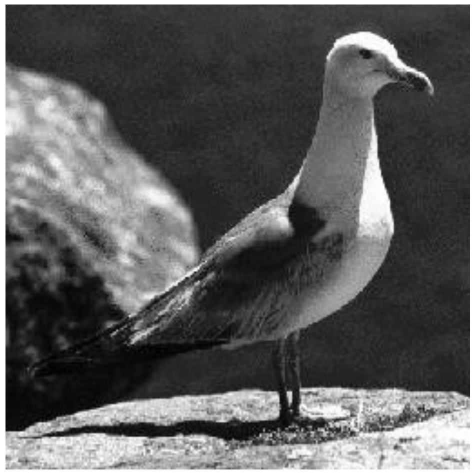
Feature is invariant to absolute intensity values

What are the problems?

How can you be less sensitive to deformations?

Image Representations: Histograms





Motivation: We want some sensitivity to spatial layout, but not too much, so blocks of histograms give us that

Global histogram to represent distribution of features

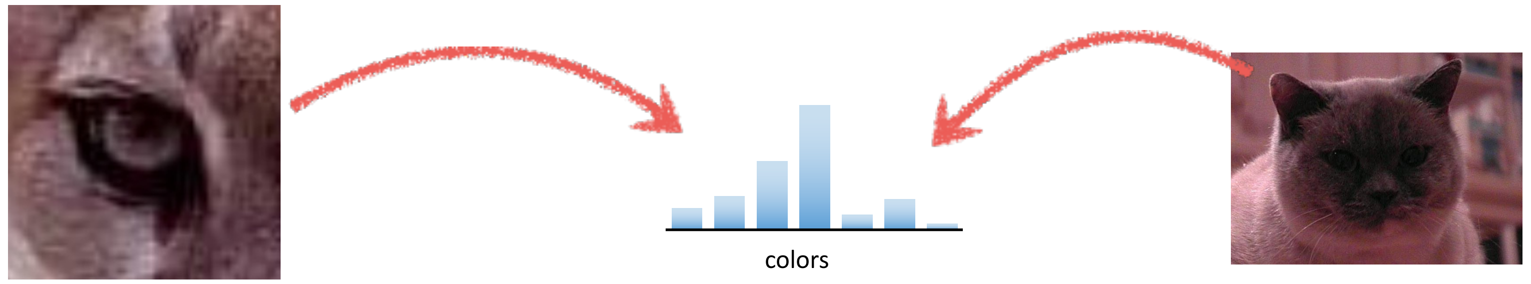
– How 'well exposed' a photo is

What about a local histogram per detected point?

Images from Dave Kauchak

Intensity/Color histogram

Count the colors in the image using a histogram



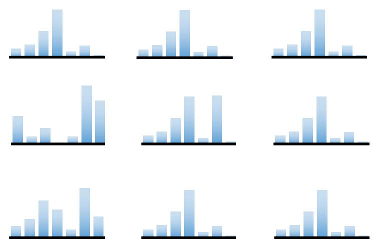
Invariant to changes in scale and rotation

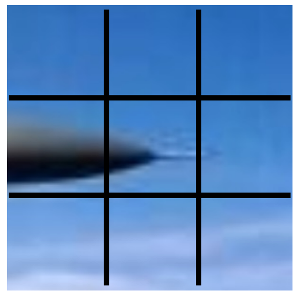
What are the problems?

How can you be more sensitive to spatial layout?

Spatial histograms

Compute histograms over spatial 'cells'





Retains rough spatial layout

Some invariance to deformations

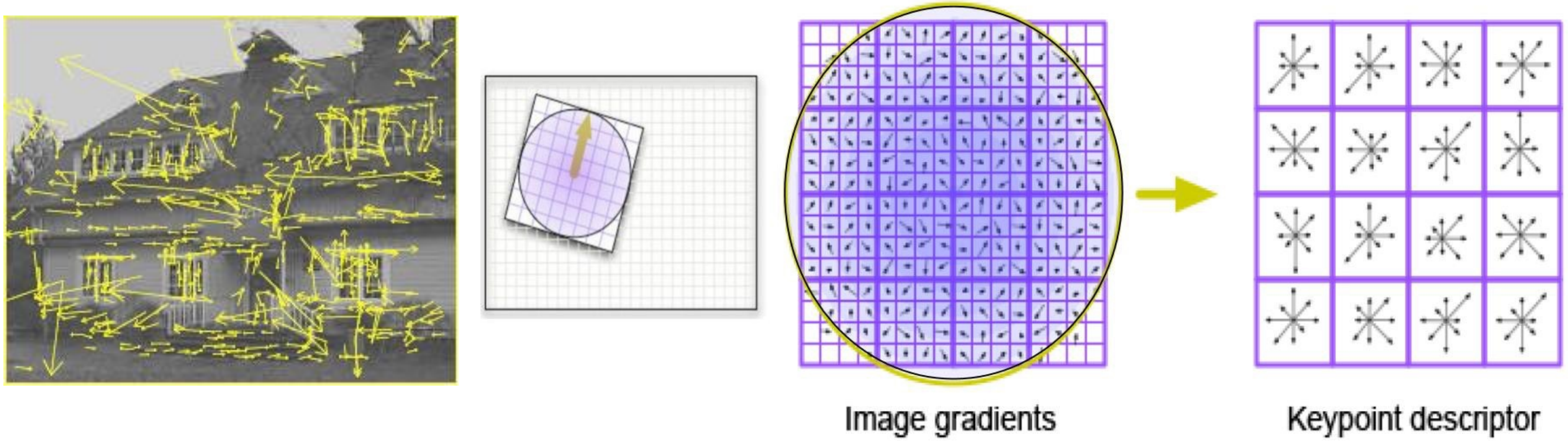
What are the problems?

How can you be completely invariant to rotation?

For what things might we compute histograms?

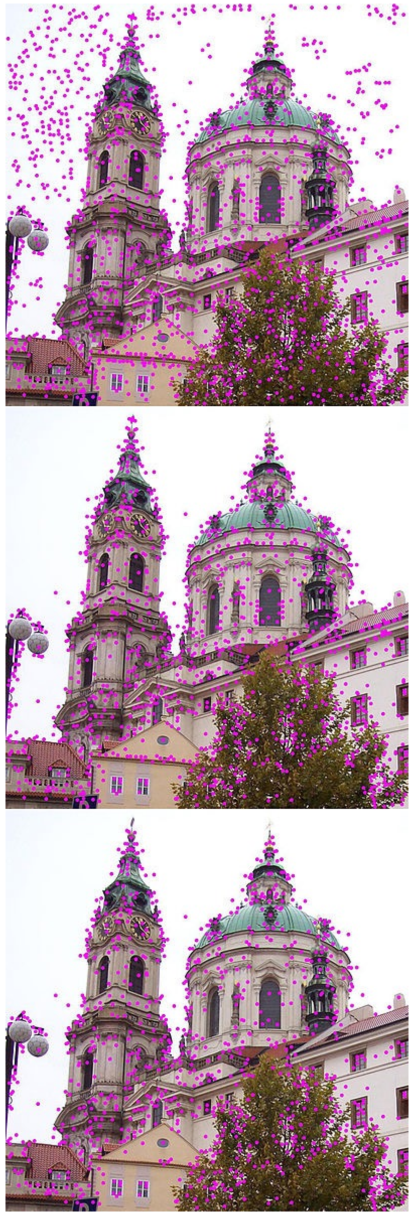
- *From Color to Texture/Keypoints*
- Histograms of oriented gradients (**HOG**)
- Scale Invariant Feature Transform (**SIFT**)
  - Extremely popular (63k citations in 2021)

IMHO, one of the most elegant designs ever in CV



SIFT – Lowe IJCV 2004

James Hays

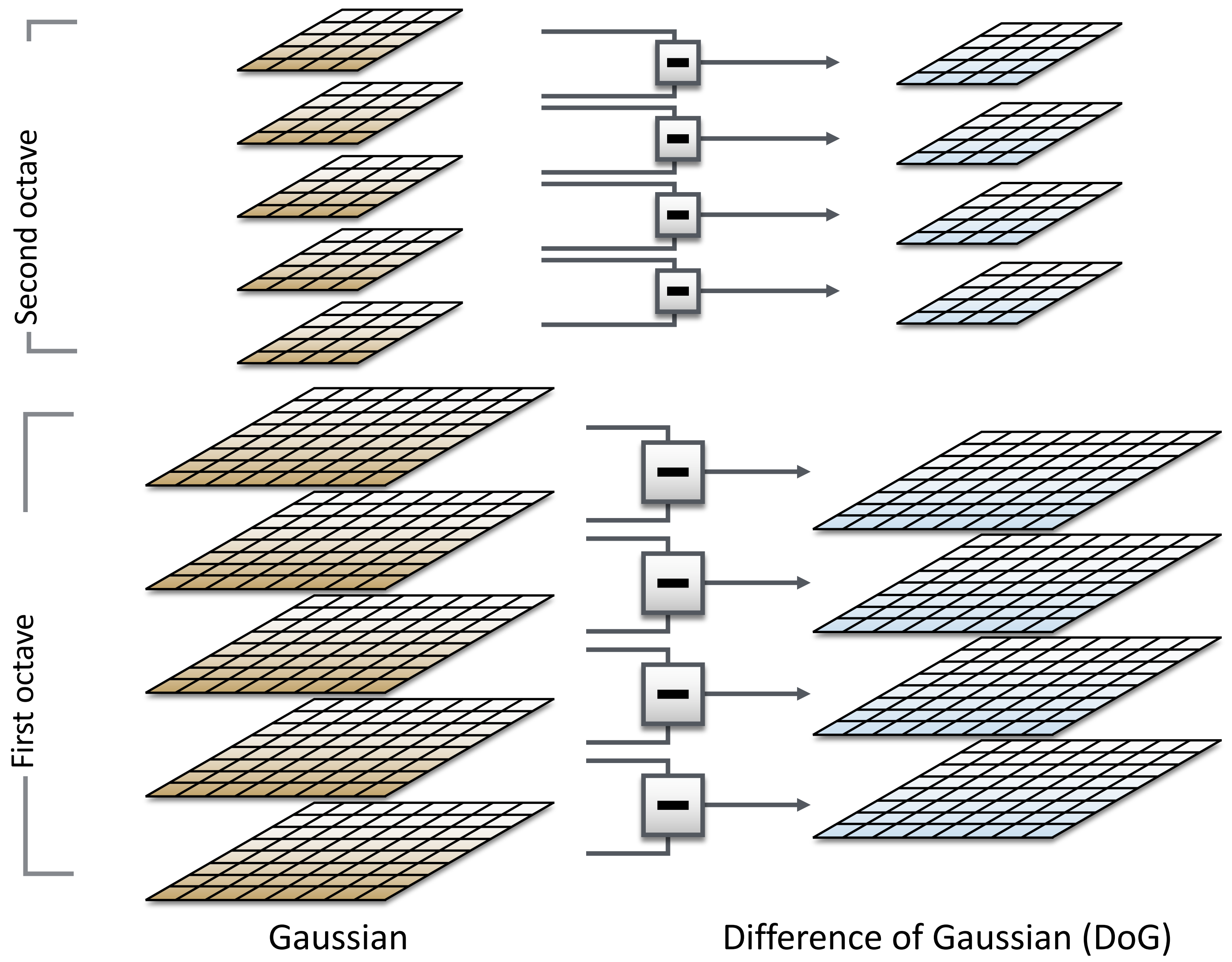


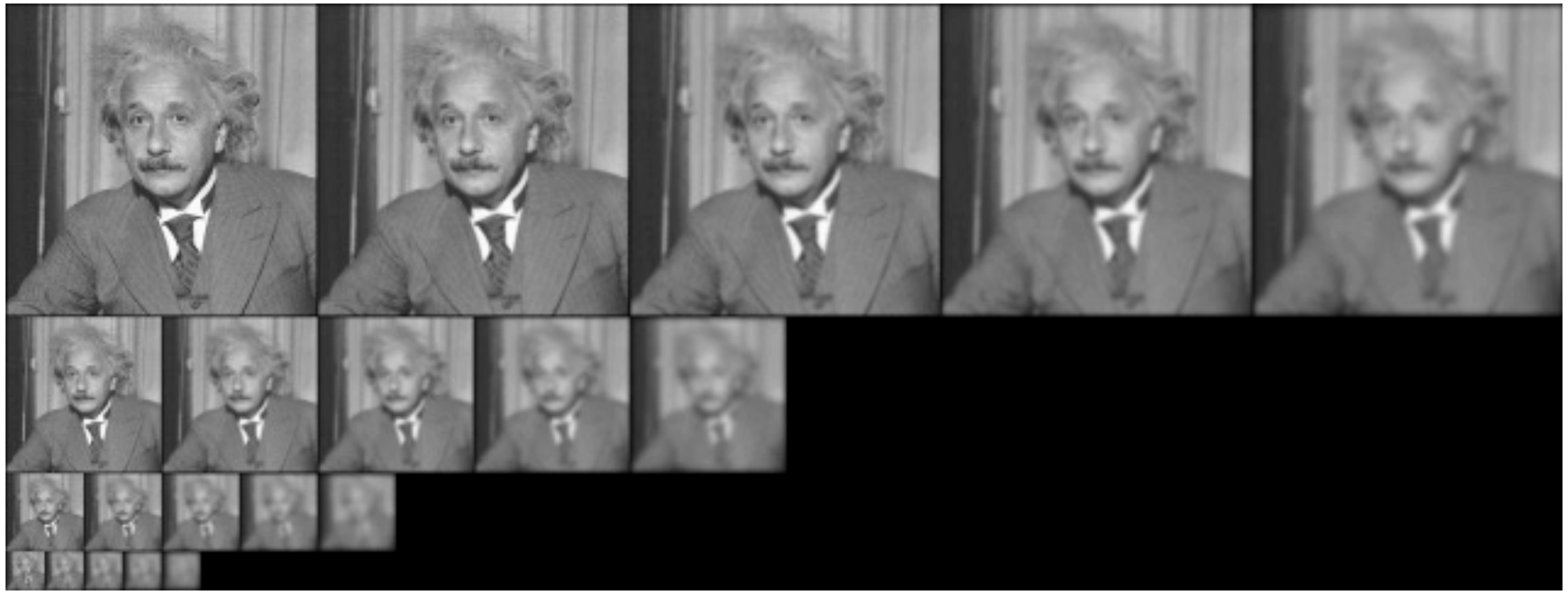
SIFT (Scale Invariant Feature Transform)

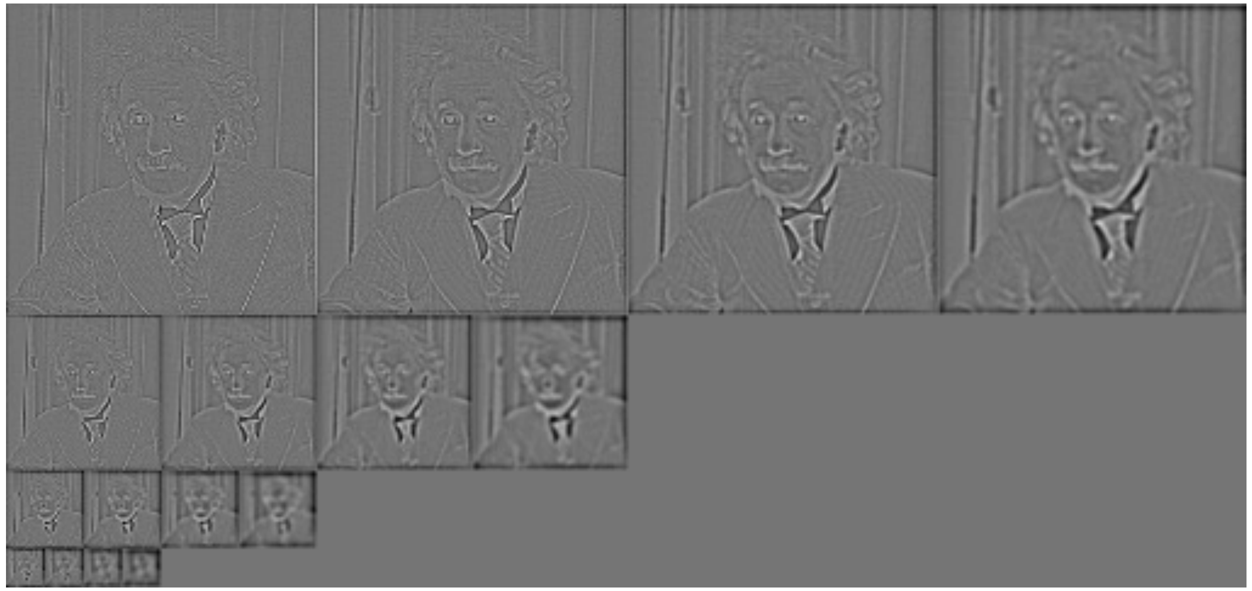
SIFT describes both a **detector** and **descriptor**

1. Multi-scale extrema detection
2. Keypoint localization
3. Orientation assignment
4. Keypoint descriptor

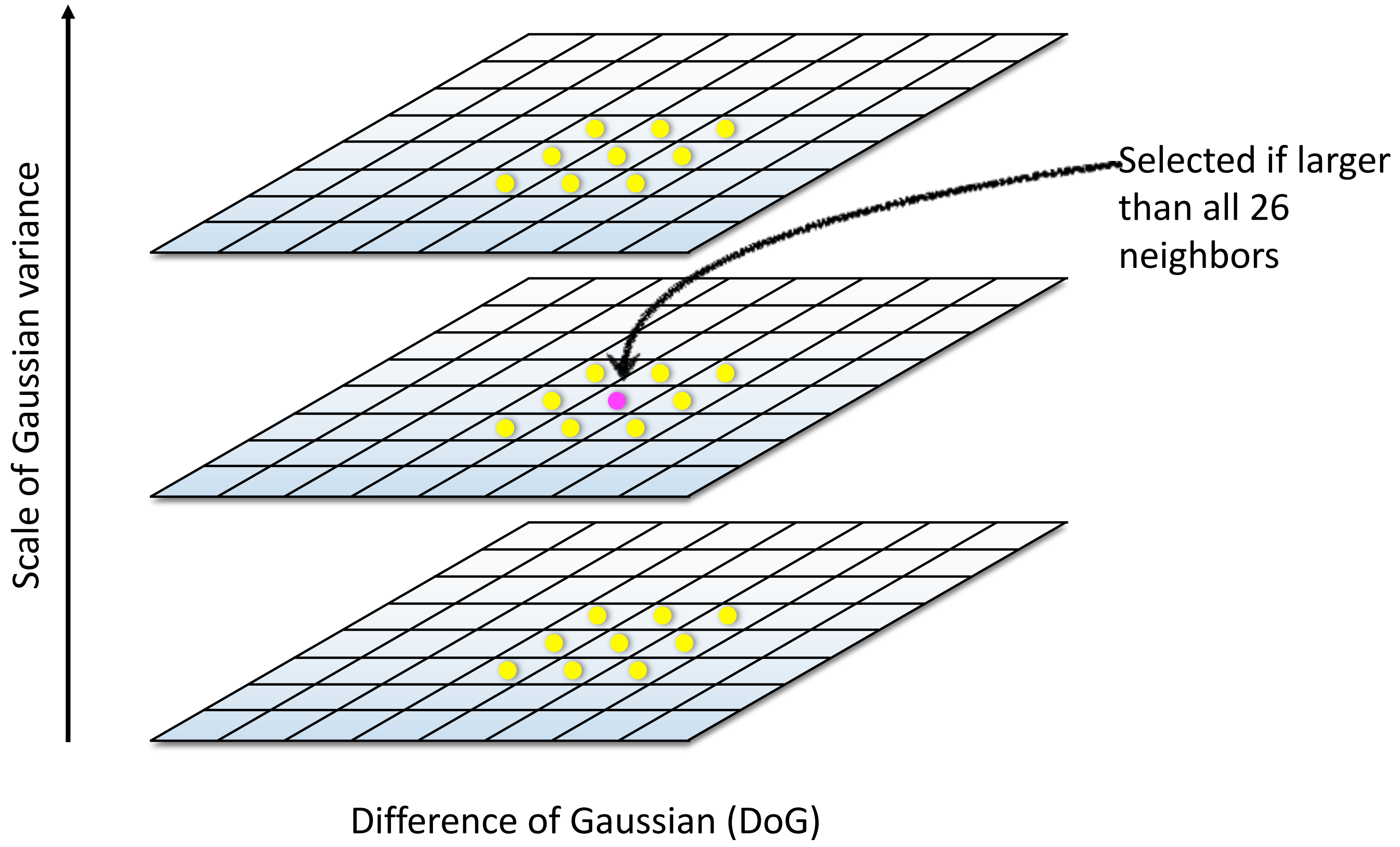
1. Multi-scale extrema detection




Gaussian


Laplacian

Scale-space extrema


Scale of Gaussian variance
Selected if larger than all 26 neighbors
Difference of Gaussian (DoG)

2. Keypoint localization (why this step?)

- Reject flats:
  - $|D(\hat{\mathbf{x}})| < 0.03$
- Reject edges:

$$\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

Let α be the eigenvalue with larger magnitude and β the smaller.

$$\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \alpha + \beta,$$

$$\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - (D_{xy})^2 = \alpha\beta.$$

Let r = α/β.
So α = rβ

$$\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\alpha+\beta)^2}{\alpha\beta} = \frac{(r\beta+\beta)^2}{r\beta^2} = \frac{(r+1)^2}{r},$$

$(r+1)^2/r$ is at a min when the 2 eigenvalues are equal.

  - r < 10

3. Orientation assignment

For a keypoint, **L** is the **Gaussian-smoothed** image at its selected scale,

$$m(x,y) = \sqrt{\underbrace{(L(x+1,y) - L(x-1,y))^2}_{\text{x-derivative}} + \underbrace{(L(x,y+1) - L(x,y-1))^2}_{\text{y-derivative}}}$$

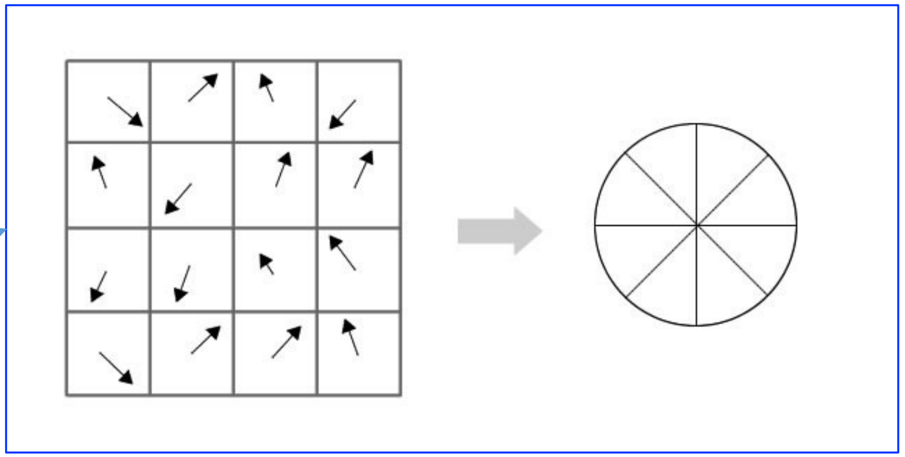
$$\theta(x,y) = \tan^{-1}((L(x,y+1) - L(x,y-1))/(L(x+1,y) - L(x-1,y)))$$

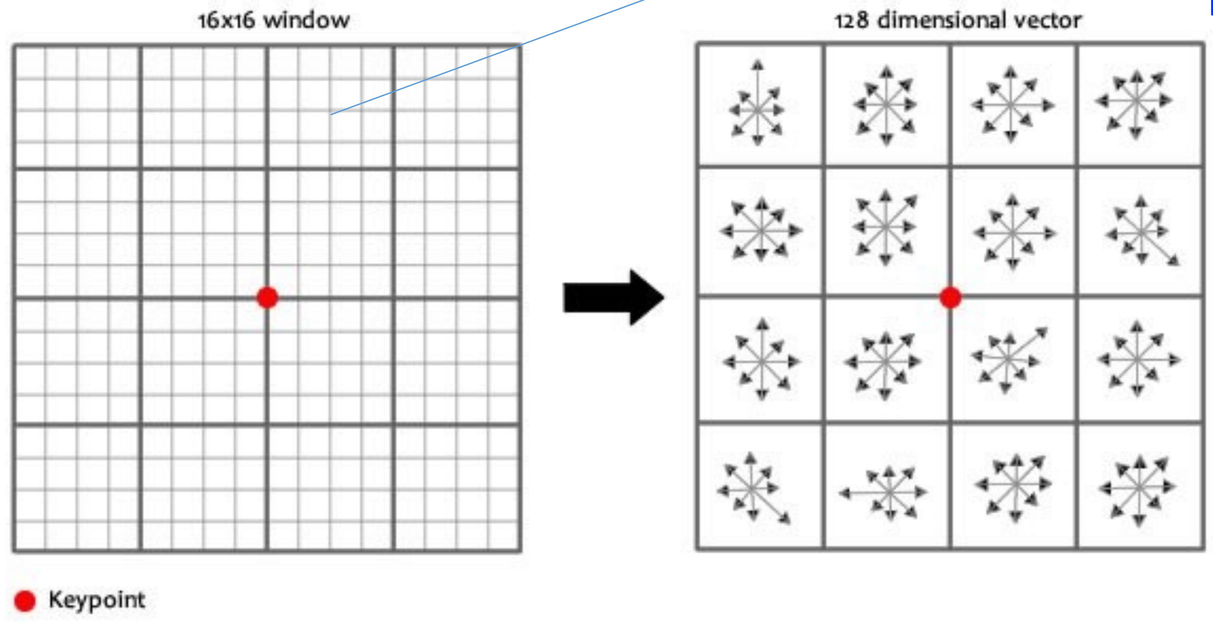
Detection process returns $\{\underbrace{x, y}_{\text{location}}, \underbrace{\sigma}_{\text{scale}}, \underbrace{\theta}_{\text{orientation}}\}$

4. Keypoint descriptor

At this moment, each survived keypoint has {location, scale, orientation}…

- Use local image gradients at selected scale and rotation to describe each keypoint region.





Adding more invariances to SIFT

Rotation Invariance:

- The feature vector uses gradient orientations. So if you rotate the image, everything changes!
- SIFT adopts "relative rotation": the keypoint's own rotation is subtracted from each orientation. Thus each gradient orientation is relative to the keypoint's orientation.

Illumination Invariance:

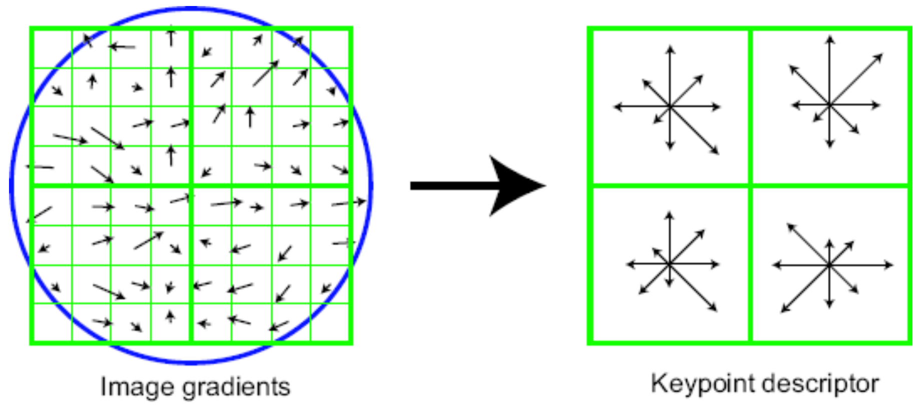
- All keypoints' 128-dim vector are normalized to 1
- Sometimes we have "outlier illumination" …
  - Practically, after normalization, we clamp all gradients > 0.2, then renormalize to [0,1]

*SIFT achieves an extremely elegant and robust balance between **global layout** (histogram) versus **local feature** (full gradient), discriminativeness versus resilience*

inliers: 687/734
TEXTS IN COMPUTER SCIENCE
Computer Vision
Algorithms and Applications
Richard Szeliski
Springer

Review: Local Descriptors

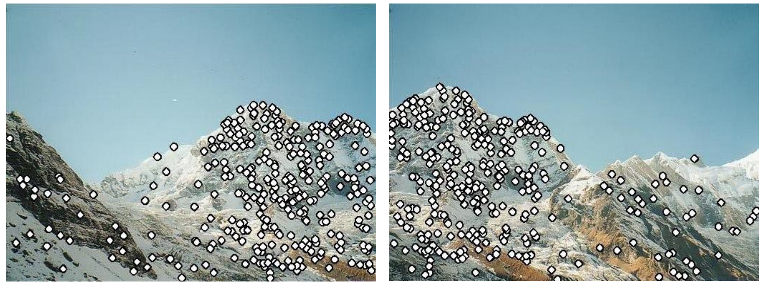
- Most features can be thought of as templates, histograms (counts), or combinations
- The ideal descriptor should be
  - Robust and Distinctive
  - Compact and Efficient
- Most available descriptors focus on edge/gradient information
  - Capture texture information
  - Color rarely used



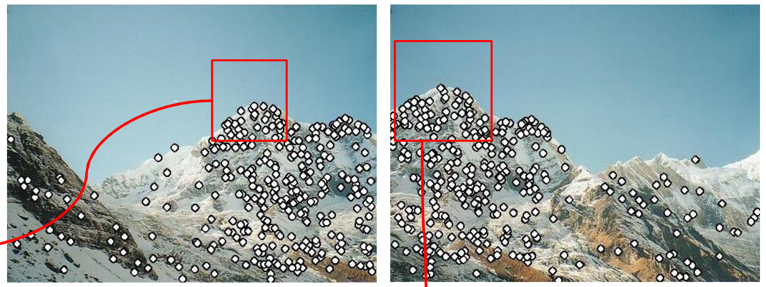
K. Grauman, B. Leibe

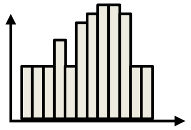
Local features: main components

1) **Detection:**
Find a set of distinctive key points.

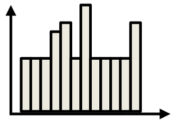


2) **Description:**
Extract feature descriptor around each interest point as vector.



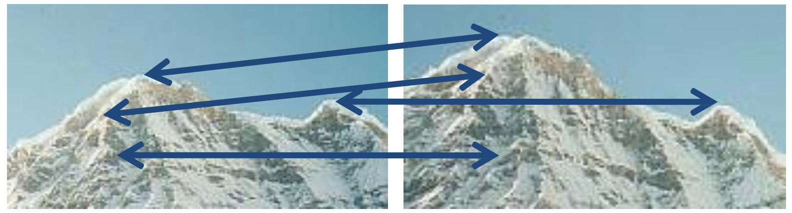
x_1 

$$\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$$

x_2 

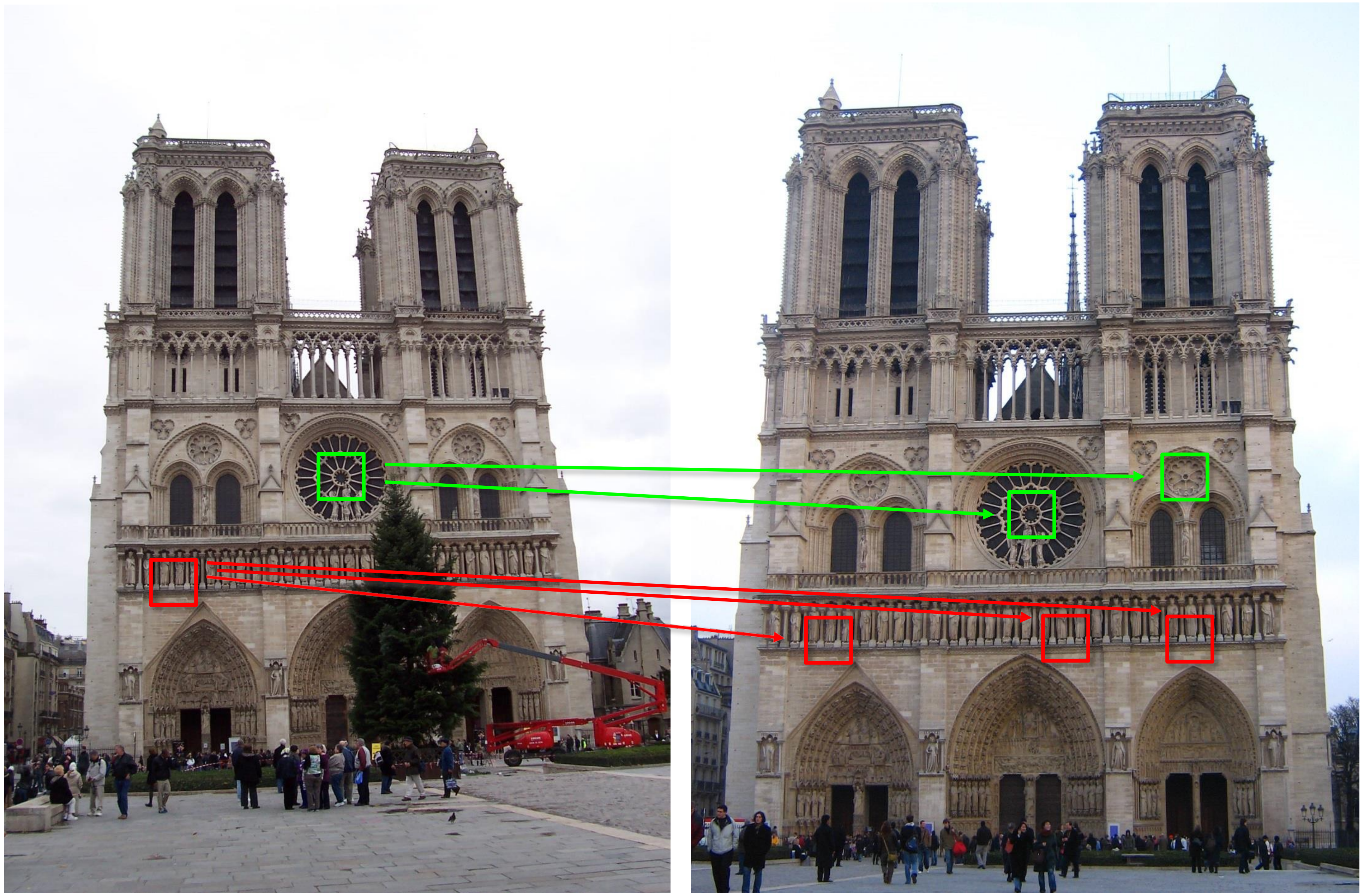
$$\mathbf{x}_2 = [x_1^{(2)}, \ldots, x_d^{(2)}]$$

3) **Matching:**
Compute distance between feature vectors to find correspondence.



Ok, now we have local features…

But how similar can the two features be called “match”?

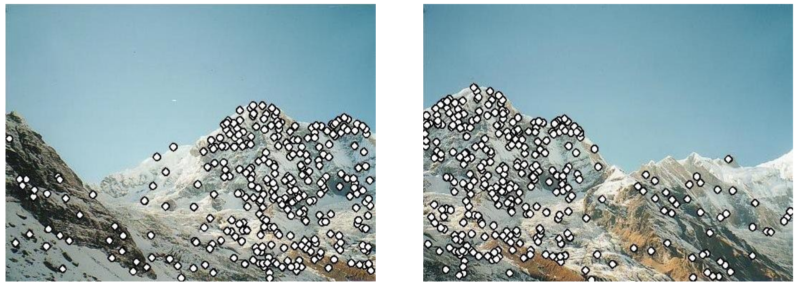


Distance: 0.34, 0.30, 0.40

Distance: 0.61, 1.22

What to Consider in the Design of Feature Matching

- Two images, I_1 and I_2



- Two sets X_1 and X_2 of feature points
  - Each feature point $\mathbf{x_1}$ has a descriptor $\mathbf{x}_1 = [x_1^{(1)}, \ldots, x_d^{(1)}]$
- Distance, bijective/injective/surjective, noise, confidence, computational complexity, generality…

Euclidean distance vs. Cosine Similarity

- Euclidean distance:

$$d(\mathbf{p},\mathbf{q}) = d(\mathbf{q},\mathbf{p}) = \sqrt{(q_1-p_1)^2 + (q_2-p_2)^2 + \cdots + (q_n-p_n)^2}$$

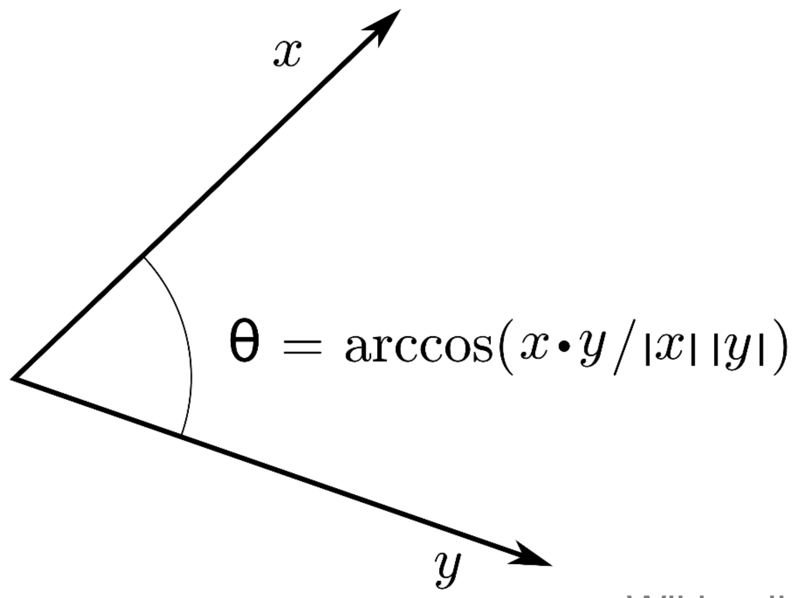
$$= \sqrt{\sum_{i=1}^{n}(q_i-p_i)^2}.$$

$$\|\mathbf{q}-\mathbf{p}\| = \sqrt{(\mathbf{q}-\mathbf{p})\cdot(\mathbf{q}-\mathbf{p})}.$$

- Cosine similarity:

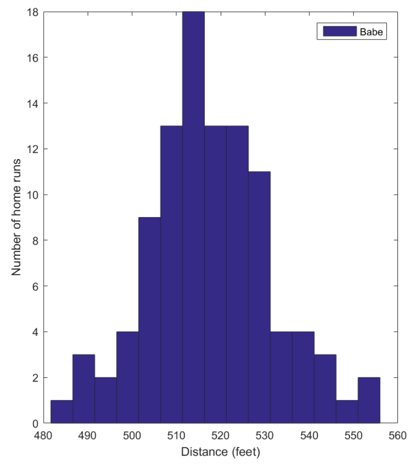
$$\mathbf{a}\cdot\mathbf{b} = \|\mathbf{a}\|_2\|\mathbf{b}\|_2\cos\theta$$

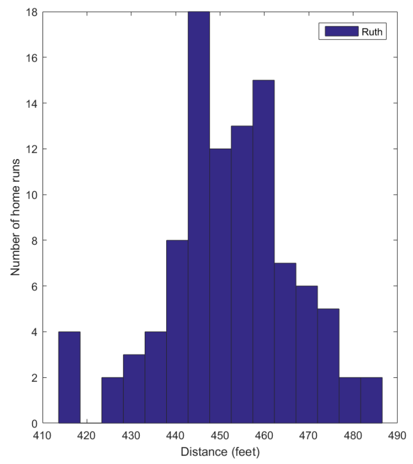
$$\text{similarity} = \cos(\theta) = \frac{\mathbf{A}\cdot\mathbf{B}}{\|\mathbf{A}\|_2\|\mathbf{B}\|_2}$$



Wikipedia

Computing histogram distance





$$\text{histint}(h_i, h_j) = 1 - \sum_{m=1}^{K} \min\left(h_i(m), h_j(m)\right)$$

Histogram intersection (assuming normalized histograms)

$$\chi^2(h_i, h_j) = \frac{1}{2} \sum_{m=1}^{K} \frac{[h_i(m) - h_j(m)]^2}{h_i(m) + h_j(m)}$$

Chi-squared Histogram matching distance

Cars found by color histogram matching using chi-squared

Histograms: Implementation issues

- Quantization
  - Grids: fast but applicable only with few dimensions
  - Clustering: slower but can quantize data in higher dimensions

Few Bins
Need less data
Coarser representation

Many Bins
Need more data
Finer representation

- Matching
  - Histogram intersection or Euclidean may be faster
  - Chi-squared often works better
  - Earth mover's distance is good for when nearby bins represent similar values

James Hays

Feature Matching

- Criteria 1:
  - Compute distance in feature space, e.g., Euclidean distance between 128-dim SIFT descriptors
  - Match point to lowest distance (nearest neighbor)

- Problems:
  - Does everything have a match?

Feature Matching

- Criteria 2:
  - Compute distance in feature space, e.g., Euclidean distance between 128-dim SIFT descriptors
  - Match point to lowest distance (nearest neighbor)
  - Ignore anything higher than threshold (no match!)

- Problems:
  - Threshold is hard to pick
  - Non-distinctive features could have lots of close matches, only one of which is correct

Nearest Neighbor Distance Ratio

Compare distance of closest (NN1) and second-closest (NN2) feature vector neighbor.

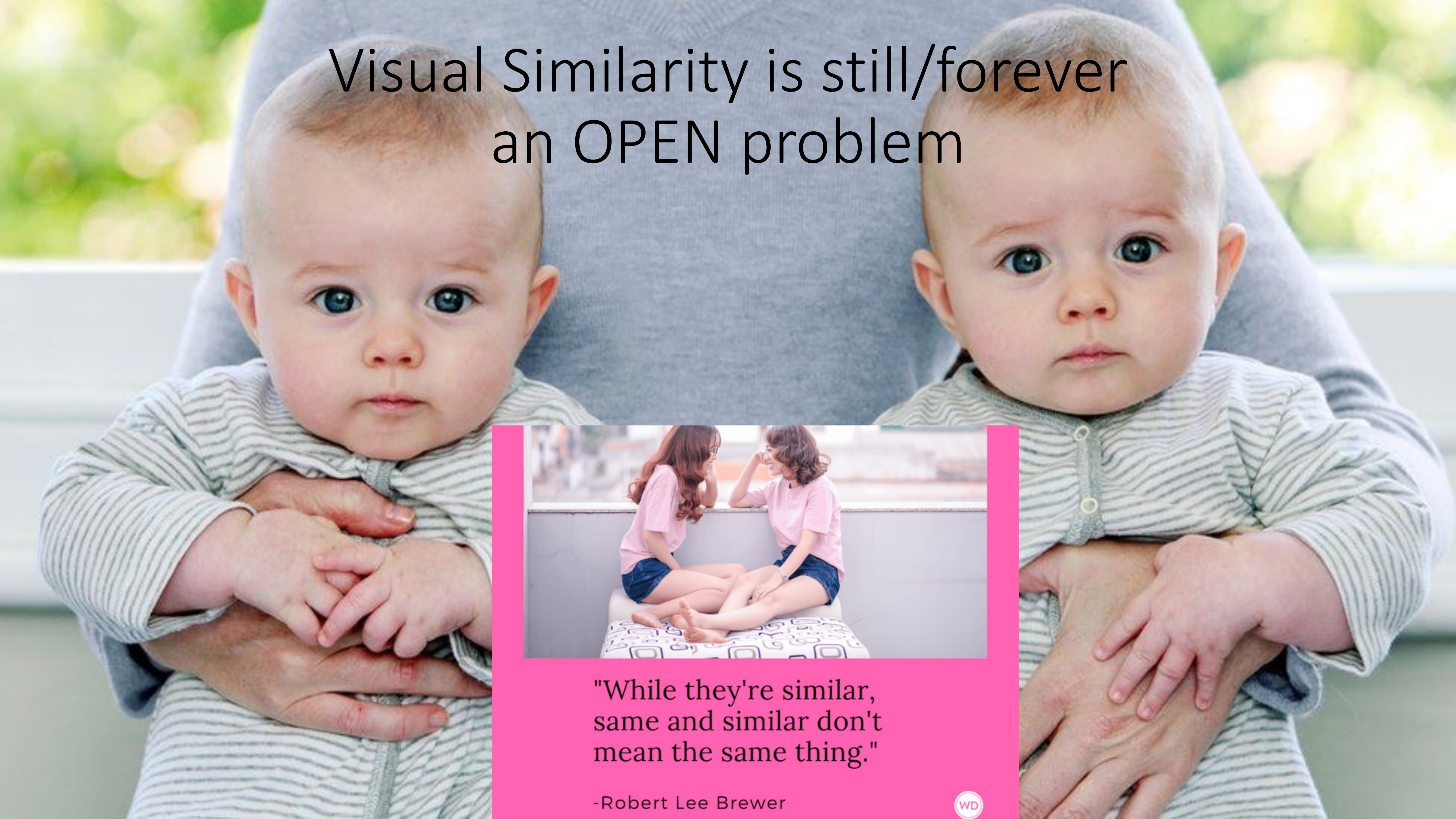
- If NN1 ≈ NN2, ratio $\frac{NN1}{NN2}$ will be ≈ 1 -> matches too close.
- As NN1 << NN2, ratio $\frac{NN1}{NN2}$ tends to 0.

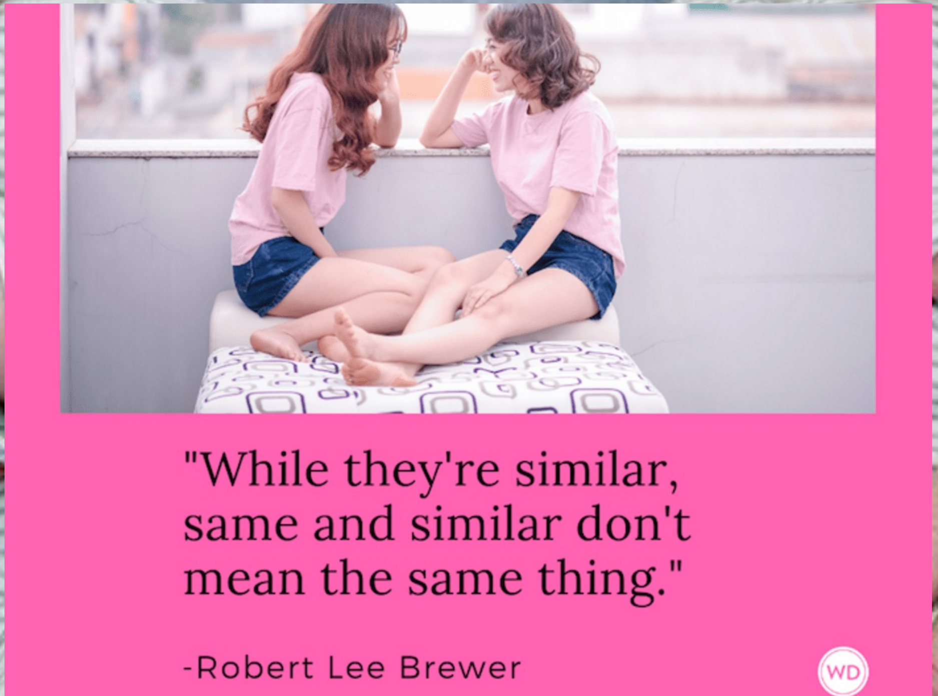
Sorting by this ratio puts matches in order of confidence.

Threshold ratio – but how to choose?

- depends on your application’s view on the trade-off between the number of false positives and true positives! You need to tune…

Visual Similarity is still/forever an OPEN problem





"While they're similar, same and similar don't mean the same thing."

-Robert Lee Brewer

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